

# UGIOMER + KLINOMER

1 JUN 2005.

• NAČI MAK. MERENI UGIB I UPOREDITI GA SA RAČUNSKIM.

( $f_m = 0,01 \text{ mm}$ )

STANJE	$U_1$	$U_5$	$U_2$	$K_1$	$K_2$
0	0,428	1,151	1,101	0+0,28	0+1,21
2	0,740	3,281	1,621	4+2,30	4+0,74
$\Delta C$	3,12	2,130	5,20	14,52	12,97 $\times 1,06''$
$\Delta C \cdot p + u_t$	3,12	2,13	5,2	15,39,12	13,74,82

$f_m = 0,01$   $p_k = 1,06''$

PODATAK KLINOMETRA  
PODATAK UGIOMERA

$J = 5740 \text{ cm}^4$

$\frac{\pi}{3600} \cdot 130$

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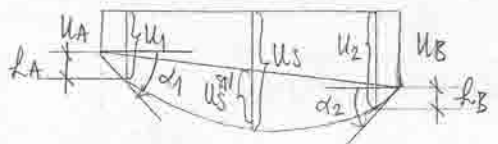
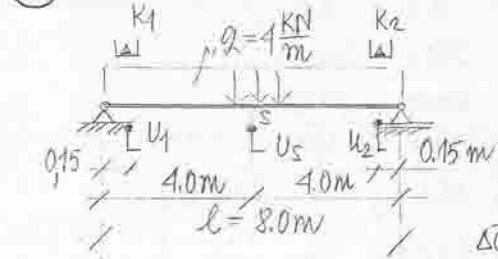
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$$U_1 = U_A + L_A \quad U_2 = U_B + L_B$$

$$\text{tg } \alpha_1 = \frac{L_A}{150} \rightarrow L_A = 1539,12 \cdot \frac{\pi}{648000} \cdot 150 = 1,12 \text{ mm}$$

$$\text{tg } \alpha_2 = \frac{L_B}{150} \rightarrow L_B = 1374,82 \cdot \frac{\pi}{648000} \cdot 150 = 1,00 \text{ mm}$$

$$U_{SN} = U_5 - \frac{U_A + U_B}{2} = 2,13 - \frac{2,0 + 4,2}{2} = 18,2 \text{ mm}$$

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$$U_A = U_1 - L_A = 3,12 - 1,12 = 2,0 \text{ mm}$$

$$U_B = U_2 - L_B = 5,2 - 1,0 = 4,2 \text{ mm}$$

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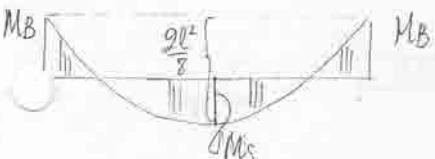
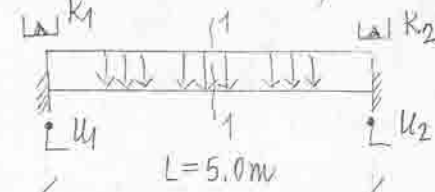
$$U_B = U_2 - L_B = 5,2 - 1,0 = 4,2 \text{ mm}$$

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2 → KOJE INSTRUMENTE ZA MERENJE DILATACIJA MOŽE MO UPOTREBITI ZA MERENJE U PRESKU U SREDINI RASPOA, A NA OSNOVU DATIH MERENJA.



MOMENTI TOTALNOG UKLOST=HA:

$$M_A = M_B = \frac{q l^2}{12} = \frac{5 \cdot 5^2}{12} = 10,42 \text{ kNm}$$

$$\epsilon_s = \epsilon \cdot E_s \rightarrow \epsilon_s = \frac{\sigma_s}{E}$$

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MOMENTI ELASTICNOG UKLOST=HA:

$$M_A = M_B = \frac{2 E J (\alpha_B - 2 \alpha_A) + M_A}{5} = \frac{2 \cdot 2,1 \cdot 2140}{5} (7,6571 \cdot 10^{-4} - 2 \cdot 7,6571 \cdot 10^{-4}) + 10,42 = 9,049 \text{ kNm}$$

$$M_S = \frac{q l^2}{8} - \frac{M_A + M_B}{2} = \frac{5 \cdot 5^2}{8} - \frac{9,049 + 9,049}{2} = 6,521 \text{ kNm}$$

$$I_{200} \rightarrow l = 20 \text{ cm} \quad W = \frac{J}{10} = 214 \text{ cm}^3 \rightarrow \sigma_s = \frac{6,521 \cdot 10^2}{214} = 3,075 \frac{\text{KN}}{\text{cm}^2} \rightarrow \epsilon_s = \frac{\sigma_s}{E} = \frac{3,075}{2,1 \cdot 10^4}$$

IZBOR INSTRUMENTA:

INSTRUMENT	BAZA	PODATAK	$\Delta C$	KOMENTAR	GRESKA OČITANJA
MERNA TRAKA	1/8 BITNA	$1 \cdot 10^{-6}$	146	ODGOVARA	0,7%
LABISKON "HUGENBERGER"	250mm	$4 \cdot 10^{-6}$	36	ODGOVARA	2,8%
LABISKON PFENDER TENZOMETAR	100mm	$10 \cdot 10^{-6}$	14	USLOVNO	7,1%
TEUZOMETAR	20mm	$50 \cdot 10^{-6}$	2,9	NE ODGOVARA	34,5%

$$\epsilon_s = 146,4 \cdot 10^{-6}$$

$$\epsilon = \frac{K \cdot \Delta C}{K_A}$$

$$\text{GRESKA} = \frac{1}{\Delta C} \times 100$$

OKO 7% J= GRANICA!

- 3) NA OSNOVU PODATAKA MERENJA IZRAČUNATI STVARNU VREDNOST MAX UGIBA I VEĆINU PROBNOS OPTEREĆENJA.

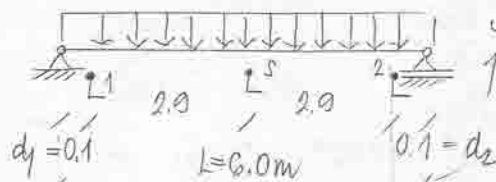
SEP. 2003

$$E_c = 2,1 \times 10^4 \frac{\text{KN}}{\text{cm}^2}$$

$$J = 9800 \text{ cm}^4$$

$$f_m = 0,01 \text{ mm}$$

ST.	$u_1$	$u_s$	$u_2$	
0	0125	0222	0368	
2	0555	2072	0748	
2-0	430	1850	380	$\times 0,01$
UGIB	4,3	18,5	3,8	[mm]



$$f_{mur} = u_s - \frac{u_1 + u_2}{2} = 18,5 - \frac{4,3 + 3,8}{2} = 18,5 - 4,05 = 14,45 \text{ mm}$$

$$Q_2 = \frac{f}{f_1} = \frac{f}{5l_1^4 + 12d_1 l_1^2(l-d_1) + 12d_2 l_1^2(l-d_2)}$$

$$Q_2 = \frac{5 \cdot 6,0^4}{5 \cdot 5,84 + 12 \cdot 0,1 \cdot 5,8^2(6-0,1) + 12 \cdot 0,1 \cdot 5,8^2(6-0,1)} = \frac{6480}{6134,5904}$$

$$Q_2 = 1,0563 \rightarrow f = 1,0563 \cdot 14,45 = 15,26 \text{ mm}$$

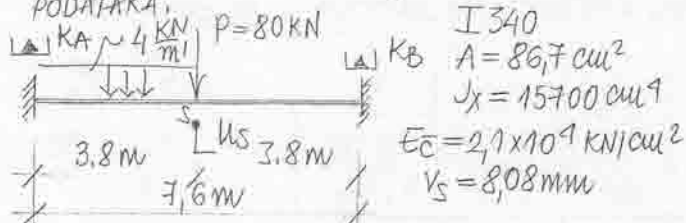
$$f_{rac} = \frac{5}{384} \frac{q_{pr} \cdot l^4}{EJ} \rightarrow q_{pr} = \frac{f \cdot EJ}{5 \cdot l^4} \cdot 384$$

$$q_{pr} = \frac{1,526 \cdot 2,1 \cdot 10^4 \cdot 9800}{5 \cdot 600^4} \cdot 384 = 0,7861 \text{ KN/cm} = 18,61 \text{ KN/m}$$

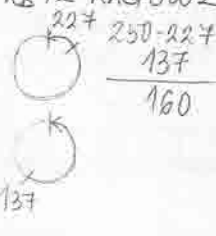
$$f_{max} = 15,26 \text{ mm} \quad q_{pr} = 18,61 \text{ KN/m}$$

MAJ 2004.

- 4) TOKOM MERENJA OPSTIH DEFORM. PROMENE NAGIBA NAD OSUONICIMA I UGIBA SREDINE GREDE IZGUBLJENO JE MERENJE NAGIBA  $\alpha_B$ . ODREDITI ČITANJE NA KLINOMETRUM KB IZ RASPOLOŽIVIH PODATAKA.



	KA	KB
0	0+227	0+072
OPT.	0+137	?+???
OPT-0	160	
$\alpha''$	169,6	
$\hat{\alpha}$	8,22210	



MOMENTI TOTALNOG UKLOSTENJA:

$$M_A = \frac{Pl}{8} + \frac{11}{192} 2l^2 = \frac{80 \cdot 3,8}{8} + \frac{11}{192} 4 \cdot 3,8^2 = 89,24 \text{ KNm}$$

$$M_B = \frac{Pl}{8} + \frac{5}{192} 2l^2 = \frac{80 \cdot 3,8}{8} + \frac{5}{192} 4 \cdot 3,8^2 = 82,02 \text{ KNm}$$

MOMENTI ČASNOG UKLOSTENJA:

$$\left. \begin{aligned} M_A &= \frac{2EJ}{l} (\alpha_B - 2\alpha_A) + M_A \\ M_B &= \frac{2EJ}{l} (\alpha_A - 2\alpha_B) + M_B \end{aligned} \right\} \begin{aligned} M_A + M_B &= \frac{2EJ}{l} (\alpha_B - 2\alpha_A + \alpha_A - 2\alpha_B) + M_A + M_B = \\ &= -\frac{2EJ}{l} (\alpha_A + \alpha_B) + M_A + M_B \end{aligned}$$

$$V_s = \frac{5}{768} \frac{2l^4}{EJ} + \frac{Pl^3}{48EJ} - \frac{1}{16} \frac{l^2}{EJ} (M_A + M_B)$$

$$EJV_s = \frac{5}{768} \cdot 2l^4 + \frac{1}{48} Pl^3 - \frac{l^2}{16} (M_A + M_B)$$

$$EJV_s = \frac{5}{768} \cdot 4 \cdot 3,8^4 + \frac{1}{48} 80 \cdot 3,8^3 - \frac{3,8^2}{16} (-8676,316\alpha_B + 169,126)$$

$$EJV_s = 31321,5\alpha_B + 226,012$$

$$2,1 \cdot 15700 \cdot 808 \cdot 10^{-3} = 31321,5\alpha_B + 226,012$$

$$\alpha_B = 1,289 \cdot 10^{-3} = \frac{\alpha'' \cdot \pi}{648000} \rightarrow \alpha'' = 265,95''$$

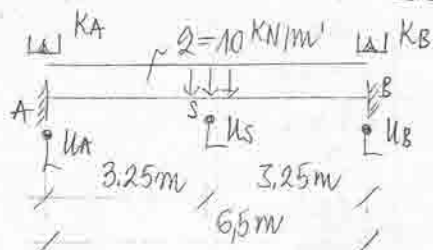
$$\Delta \bar{\alpha} = \frac{\alpha''}{1,06''} = 251$$

$$\begin{aligned} \text{KB} &\rightarrow 0: 0+072 \\ &2: 0+71 \end{aligned}$$

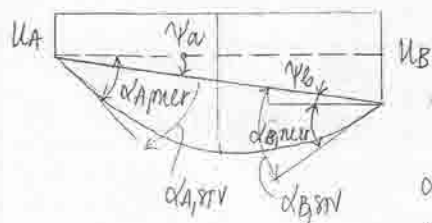
$$2-0: 251$$

$$\begin{aligned} 72+250-X &= 251 \\ X &= 71 \end{aligned}$$

5) JUN 2'02. - NAOI STVARH= OSLOVAČKE MOMENTE  $M_A$  I  $M_B$ , KAO I STEPENE UKLJEŠTENJA I UPOREDITI IH SA RAČUNSKIM.  $\rho_m = 0,01 \text{ mm}$ ,  $I = 240 \dots$ ,  $J_x = 4250 \text{ cm}^4$ ,  $E = 21 \times 10^4 \text{ KN/cm}^2$



ST	UA	US	UB	K1	K2
0	0,158	0,025	0,223	0,1023	0,1055
2	0,258	1,099	0,823	0,1123	0,1173
2-0	100	1074	600	350	132
ut.	1,0	1074	6,0	371	134,92



$$\alpha_{A,mer} = 1,798 \cdot 10^{-3} \text{ rad}$$

$$\alpha_{B,mer} = 6,784 \cdot 10^{-4} \text{ rad}$$

$$v_{AB} = \frac{U_B - U_A}{l} = \frac{(6,0 - 1,0) \cdot 10^{-3}}{6,5} = 7,692 \cdot 10^{-4} \text{ rad}$$

$$\alpha_{A,sn} = \alpha_{A,mer} - v_{AB} = 1,798 \cdot 10^{-3} - 7,692 \cdot 10^{-4} = 1,0288 \cdot 10^{-3}$$

$$\alpha_{B,sn} = \alpha_{B,mer} + v_{AB} = 6,784 \cdot 10^{-4} + 7,692 \cdot 10^{-4} = 1,4476 \cdot 10^{-3}$$

MOMENTI TOTALNOG UKLJEŠTENJA:

$$M_A = M_B = \frac{q l^2}{12} = \frac{10 \cdot 6,5^2}{12} = 35,208 \text{ kNm}$$

MOMENTI STVARNOG UKLJEŠTENJA:

$$M_A = \frac{2EJ}{l} [(\alpha_{B,sn} - 2\alpha_{A,sn}) + 3v_{AB}] + M_A = \frac{2 \cdot 21 \cdot 4250}{6,5} [(1,4476 \cdot 10^{-3} - 2 \cdot 1,0288 \cdot 10^{-3}) + 3 \cdot 7,692 \cdot 10^{-4}] + 35,208 \text{ kNm}$$

$$M_A = 39,870 \text{ kNm}$$

$$M_B = \frac{2EJ}{l} [(\alpha_{A,sn} - 2\alpha_{B,sn}) - 3v_{AB}] + M_B =$$

$$= \frac{2 \cdot 21 \cdot 4250}{6,5} [(1,0288 \cdot 10^{-3} - 2 \cdot 1,4476 \cdot 10^{-3}) - 3 \cdot 7,692 \cdot 10^{-4}] + 35,208 \quad M_B = 23,747 \text{ kNm}$$

OBRTANJE UKLJEŠTENJA:

$$\alpha_{A,rac} = \alpha_{B,rac} = \frac{114,427}{EI} = 0,01282096 \quad \rightarrow \text{HP} = \frac{2 \cdot l^3}{27}$$

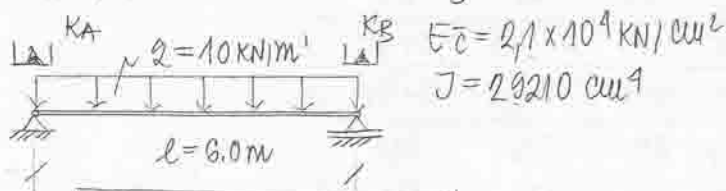
STEPENI UKLJEŠTENJA:

$$\eta_A = (1 - \frac{\alpha_{A,sn}}{\alpha_{A,rac}}) \cdot 100\% = (1 - \frac{1,0288 \cdot 10^{-3}}{0,01282096}) \cdot 100\% = 91,97\%$$

$$\eta_B = (1 - \frac{\alpha_{B,sn}}{\alpha_{B,rac}}) \cdot 100\% = (1 - \frac{1,4476 \cdot 10^{-3}}{0,01282096}) \cdot 100\% = 88,71\%$$

⑥ D=C=MEAR 2003.

ODREDITI UGLOVE OBRATA PROSTE GREDE SA OPT. PREMA SKICI IZ CIFRANJA NA KUINOMETRIMA. STA N=DOSAD=DATUM MERENJA?



$$\alpha_{A,mer} = 3,029 \cdot 10^{-3} \text{ rad}$$

$$\alpha_{B,mer} = 1,490 \cdot 10^{-3} \text{ rad}$$

$$\alpha_{A,rač} = \alpha_{B,rač} = 1,467 \cdot 10^{-3} \text{ rad} \rightarrow \text{HP}$$

ST	KA	KB
0 <sub>1</sub>	01180	01049
2	27020	07090
0 <sub>2</sub>	27180	07051
2-0 <sub>1</sub>	590	291
2-0 <sub>2</sub>	589	289
2-0 <sub>sr</sub>	589,5	290
$\alpha''$	648,87	307,41
$\alpha$	$3,029 \cdot 10^{-3}$	$1,490 \cdot 10^{-3}$

$\times 1,06'' = p\pi$

[rad]

$\alpha_{rač} \neq \alpha_{mer} \rightarrow$  POSTOJI NE-RATNO MERNO SLEGAJ= OSLONACA PATREBA POSTAVITI UGLOMERE.

$$\alpha_A = f/\beta \quad \alpha_A + \alpha_B = 2f \quad \tan \beta = \frac{u_B - u_A}{L} \rightarrow u_B - u_A = L \tan \beta$$

$$\alpha_B = f + \beta$$

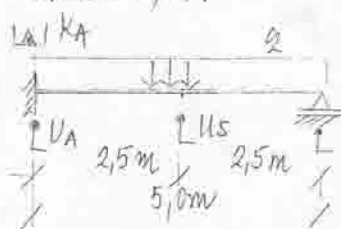
$$(3,029 - 1,490) \cdot 10^{-3} = 2f \rightarrow f = 0,0007695 \text{ rad}$$

$$3,029 \cdot 10^{-3} = 0,0007695 - \beta \rightarrow \beta = 0,0024395 \text{ rad} = 503,183''$$

$$u_B - u_A = 6 \cdot 0,0024395 = 0,014637 \text{ m} = 14,637 \text{ mm}$$

$$u_B - u_A = 14,637 \text{ mm}$$

⑦ NA Č=LIČNOJ GREDI IZVRŠENA SU MERENJA PREMA SKICI OPŠTI DEFORMACIJA. ODREDITI VELICINU PROBNOG OPT.



I 200

$$A = 33,4 \text{ cm}^2$$

$$I = 2140 \text{ cm}^4$$

$$W = 214 \text{ cm}^3$$

$$E = 2,1 \cdot 10^4 \text{ kN/cm}^2$$

$$p_m = 0,01 \text{ mm}$$

STANJE	uA	uS	uB	KA
0	0156	0243	0102	01240
2	0256	0811	0202	07060
$\Delta C$	100	568	100	70
uT	1,0	5,68	1,0	74,2''



$$u_{S,ST} = u_{S,mer} - \frac{u_A + u_B}{2} = 5,68 - 1,0 = 4,68 \text{ mm}$$

$$MOM=KAT=LAST. UKLO=ST=NA : M_A = -\frac{3EJ}{L} \alpha_A + \frac{qL^2}{8} = -\frac{3 \cdot 2,1 \cdot 2140}{5,0} \cdot 3,5973 \cdot 10^{-4} + 3,1252$$

$$3,1252 - 0,970$$

$$0,5(3,1252 - 0,970)$$

$$M_A = 3,1252 - 0,970$$

(M<sub>A</sub>)

$$EJ u_S = \int M \bar{m} ds \quad M = M_A + M_q$$

$$EJ u_S = \int M_A \bar{m} ds + \int M_q \bar{m} ds$$

$$EJ u_S = \frac{2,5}{6} \cdot 1,25(3,1252 - 1,94) - \frac{2,5}{3} \cdot 1,25 \cdot 0,5(3,1252 - 0,970)$$

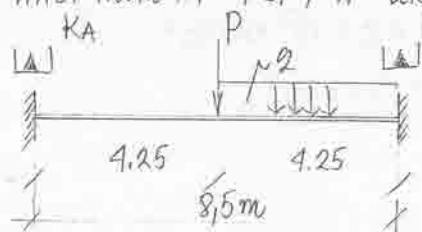
$$+ \frac{2,5}{3} \cdot 3,1252 \cdot 1,250 \cdot 2 + \frac{2,5}{3} \cdot 1,25 \cdot 0,781252 \cdot 2$$

$$= 3,25522 + 1,5156$$

$$EJ u_S = 2,1 \cdot 2140 \cdot 4,68 \cdot 10^{-3} = 21,03192 \rightarrow q = 6,0 \text{ kN/m}$$

1) S=PT. 200J.

NAČI MOMENTI I STEPENI UKLOŠTENJA GRAD PRIKAZATI NA SKICI.



$J = 29210 \text{ cm}^4$   
 $E_c = 21 \times 10^4 \text{ kN/cm}^2$   
 $P = 100 \text{ kN}$   
 $q = 4,0 \text{ kN/m}$

ST.	KA	KB
0	0+123	0+080
P	0+154	0+040
P-0	281	291
$\alpha''$	297,86	308,46
$\bar{\alpha}$	$1,444 \cdot 10^{-3}$	$1,495 \cdot 10^{-3}$

WTO 291



MOMENTI TOTALNOG UKLOŠTENJA:

$$M_A = \frac{Pl}{8} + \frac{5}{192} q l^2 = 113,776 \text{ kNm}$$

$$M_B = \frac{Pl}{8} + \frac{11}{192} q l^2 = 122,807 \text{ kNm}$$

MOMENTI ELASTIČNOG UKLOŠTENJA:

$$M_A = \frac{2EJ}{l} (\alpha_B - 2\alpha_A) + M_A = \frac{2 \cdot 21 \cdot 29210}{8,5} (1,495 \cdot 10^{-3} - 2 \cdot 1,444 \cdot 10^{-3}) + 113,776 = 93,71 \text{ kNm}$$

$$M_B = \frac{2EJ}{l} (\alpha_A - 2\alpha_B) + M_B = \frac{2 \cdot 21 \cdot 29210}{8,5} (1,444 \cdot 10^{-3} - 2 \cdot 1,495 \cdot 10^{-3}) + 122,807 = 100,58$$

OBRTANJE UKLOŠTENJA

$$\alpha_{A, \text{rac}} = \frac{1}{EJ} \left( \frac{Pl^2}{16} + \frac{7}{384} q l^3 \right) = \frac{1}{21 \cdot 29210} \left( \frac{100 \cdot 8,5^2}{16} + \frac{7}{384} \cdot 4 \cdot 8,5^3 \right) = 0,00809 \text{ rad}$$

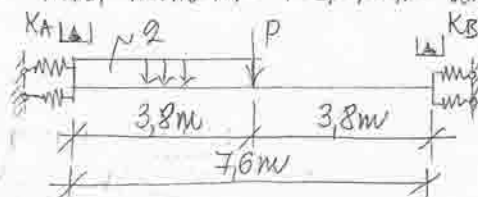
$$\alpha_{B, \text{rac}} = \frac{1}{EJ} \left( \frac{Pl^2}{16} + \frac{3}{128} q l^3 \right) = \frac{1}{21 \cdot 29210} \left( \frac{100 \cdot 8,5^2}{16} + \frac{3}{128} \cdot 4 \cdot 8,5^3 \right) = 0,00830 \text{ rad}$$

STEPENI UKLOŠTENJA

$$\eta_A = \left( 1 - \frac{\alpha_{A, \text{mer}}}{\alpha_{A, \text{rac}}} \right) \cdot 100\% = \left( 1 - \frac{1,444 \cdot 10^{-3}}{0,00809} \right) \cdot 100 = 82,20\%$$

$$\eta_B = \left( 1 - \frac{\alpha_{B, \text{mer}}}{\alpha_{B, \text{rac}}} \right) \cdot 100\% = \left( 1 - \frac{1,495 \cdot 10^{-3}}{0,00830} \right) \cdot 100 = 82,05\%$$

2) - NAČI MOMENTI I STEPENI UKLOŠTENJA:



ST.	KA	KB
0	0+229	0+072
OPT.	0+255	0+249
$\Delta$	76	73
$\alpha''$	80,56	77,38
$\bar{\alpha}$	$3,906 \cdot 10^{-4}$	$3,751 \cdot 10^{-4}$

$P = 80 \text{ kN}$

$q = 40 \text{ kN/m}$

$J = 15700 \text{ cm}^4$

$E_c = 21 \cdot 10^4 \text{ kN/cm}^2$

MOMENTI TOT. UKLOŠTENJA:

$$M_A = \frac{Pl}{8} + \frac{11}{192} q l^2 = 89,237 \text{ kNm}$$

$$M_B = \frac{Pl}{8} + \frac{5}{192} q l^2 = 82,017 \text{ kNm}$$

OBRTANJE UKLOŠTENJA:

$$\alpha_{A, \text{rac}} = \frac{1}{EJ} \left( \frac{Pl^2}{16} + \frac{3}{128} q l^3 \right) = 0,01001$$

$$\alpha_{B, \text{rac}} = \frac{1}{EJ} \left( \frac{Pl^2}{16} + \frac{7}{384} q l^3 \right) = 0,00573$$

STEPENI ELAST. UKLOŠTENJA:

$$\eta_A = \left( 1 - \frac{\alpha_{A, \text{mer}}}{\alpha_{A, \text{rac}}} \right) \cdot 100\% = 96,1\%$$

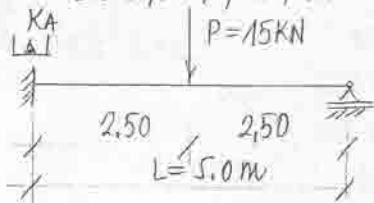
$$\eta_B = \left( 1 - \frac{\alpha_{B, \text{mer}}}{\alpha_{B, \text{rac}}} \right) \cdot 100\% = 96,15\%$$

MOMENTI ELASTIČNOG UKLOŠTENJA:

$$M_A = \frac{2EJ}{l} (\alpha_B - 2\alpha_A) + M_A = 85,72 \text{ kNm}$$

$$M_B = \frac{2EJ}{l} (\alpha_A - 2\alpha_B) + M_B = 78,89 \text{ kNm}$$

- 3 OKT. 2003 (APS)  
 - ODRĘDZIŁ SIĘ PEŁNĄ MOMENTEM  $\equiv L$ , UKŁÓSIŁ SIĘ NA I OŚ W OŚNIE A.



ST.	KA
0	0+073
P	07237
0	07071
$\Delta_1$	164
$\Delta_2$	166
$\Delta_{\text{CSR}}$	165
$\alpha''$	174,9
$\alpha$	$8,47939 \cdot 10^{-9}$

$$E_c = 2,1 \cdot 10^4 \text{ kN/cm}^2$$

$$J = 3060 \text{ cm}^4$$

MOMENT TOTALNOŚ UKŁÓSIŁ SIĘ NA I:

$$m_A = \frac{3PL}{16} = 14,0625 \text{ kNm}$$

MOMENT  $\equiv L$ , UKŁÓSIŁ SIĘ NA I:

$$M_A = -\frac{3EI}{L} \alpha_A + m_A = 10,793 \text{ kNm}$$

OBROTANIE UKŁÓSIŁ SIĘ NA I:

$$\alpha_{A, \text{rac}} = \frac{1}{EI} \frac{PL^2}{16} = 3,6473 \cdot 10^{-3}$$

SIĘ PEŁNĄ  $\equiv L$ , UKŁÓSIŁ SIĘ NA I:

$$\eta_A = \left(1 - \frac{\alpha_{\text{mer}}}{\alpha_{A, \text{rac}}}\right) 100\% = 76,76\%$$

$\times 1,06''$

4

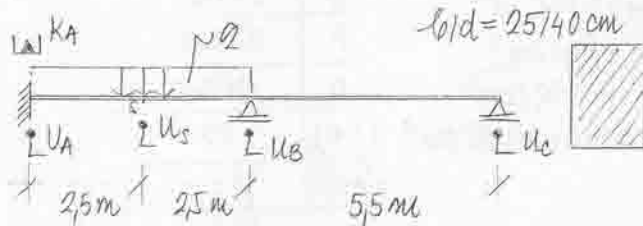


\* HOSAO PREMA SKICI ISPITIVANJE PROBNIM OPT. POTREBNO JE ODREDITI:

A) VEUCINU PROBNOG OPT. 2)

B) MOMENT IJASTICNOST UKLOSTENJA NA 1 STEPEN UKLOSTENJA

C) KOJI MERENI PODATAK NE DOSTAJE DA BISE TRAZEN VEUCINE ODREDI LAKSE I TACNU.



$$b \cdot d = 25/40 \text{ cm}$$

ST	NA	UB	US	UC	KA
0	0.464	0.237	0.461	0.359	0.245
2	0.464	0.237	0.804	0.359	0.7010
ΔC	0	0	3.43	0	15
UT.	0	0	0.343	0	15.9

250-30  
10  
10

$$U_k = 1000$$

$$E_b = 0.35 \times 10^4 \text{ kN/cm}^2$$

$$r_k = 0.001 \text{ mm}$$

$$p_k = 1.06''$$

$$J_b = \frac{1}{12} 0.25 \cdot 0.4^3 = 1.3 \cdot 10^{-3} \text{ cm}^4$$

$$E_b J_b = 46666.6 \text{ kNm}^2$$

$$E J \delta_{11} = 1.6 \quad E J \delta_{10} = 5.20832$$

$$E J \delta_{12} = 0.83 \quad -46666.6 \cdot (-1.0 \cdot 0.077) \cdot 10^{-3}$$

$$E J \delta_{22} = 3.5 \quad E \delta_{10} = 5.20832 + 3.593$$

$$E \delta_{20} = 5.20832$$

$$1.6 X_1 + 0.83 X_2 = 5.20832 - 3.593$$

$$0.83 X_1 + 3.5 X_2 = 5.20832 \quad / \cdot (-2)$$

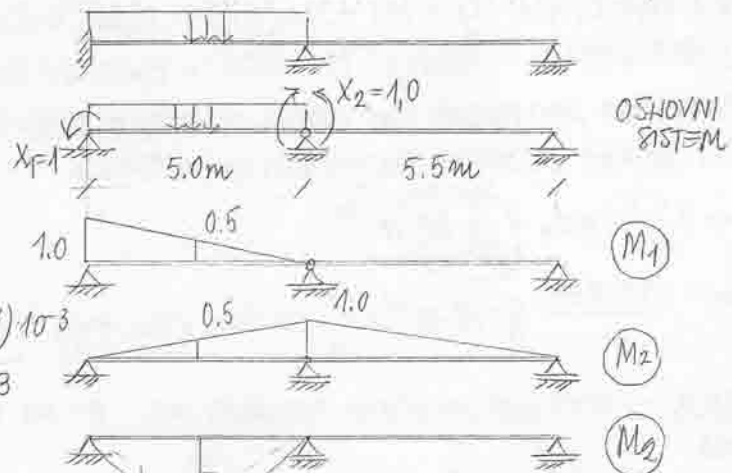
$$1.6 X_1 + 0.83 X_2 = 5.20832 - 3.593$$

$$-1.6 X_1 - 7 X_2 = -10.4162$$

$$-6.16 X_2 = -5.20832 - 3.593$$

$$X_2 = 0.8452 + 0.583$$

$$X_1 = 2.7012 - 2.449$$



$$E J U_s = \int M \bar{M} ds = \int \bar{M} (M_2 + X_1 M_1 + X_2 M_2) ds$$

$$0.8452 + 0.583$$

$$2.7012 - 2.449 - 1.3522 - 0.933$$

$$E J U_s = 2.5972 + 2.916$$

$$\rightarrow 46666.6 \cdot 0.343 \cdot 10^{-3} = 2.5972 + 2.916$$

$$q = 5.041 \text{ kN/m}$$

$$M_{A,pl} = X_1 = 2.701 \cdot 5.041 - 2.449 = 11.17 \text{ kNm}$$

$$\alpha_{A,rad} = \frac{q l^3}{24 E I}$$

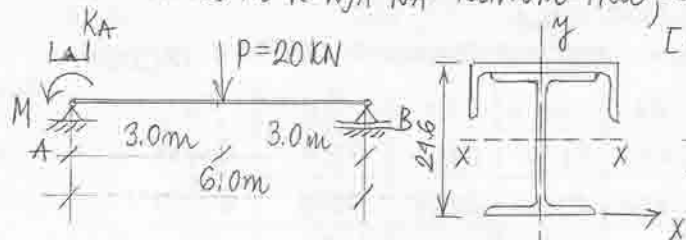
$$\alpha_{A,rad} = \frac{5.041 \cdot 5^3}{24 \cdot 46666.6} = 5.6261 \cdot 10^{-4} \text{ rad}$$

$$\eta = \left( 1 - \frac{0.077 \cdot 10^{-3}}{5.6261 \cdot 10^{-4}} \right) 100\% = 86.29\%$$

\* UPOTREBOM KUNOM TRA NAD OSLONCEM II  
MNOGO LAKSE SE DOLAZI DO PODATAKA O  
UKLOSTENJU STAPA AB.

(\*) MAJ 2003

HA OSNOVU  $M=R=N_A$  NA KLINOM=TRU, ODR=ITI M KOJIM JE OPT=R=ČEN DATI NOSAČ.



$I 200: c=201 \text{ cm}$   
 $I_{yy}=148 \text{ cm}^4$   
 $A=32.2 \text{ cm}^2$   
 $I 240: A=46.1 \text{ cm}^2$   
 $I_{x-x}=4250 \text{ cm}^4$

ST.	KA
0	0+060
P	0+249
0	0+066
(P-0) <sub>1</sub>	189
(P-0) <sub>2</sub>	183
$\Delta_{sr}$	18.6 $\times 1,06^4$
$\alpha_A''$	197,16"
$\alpha_A$	$9,558 \cdot 10^{-4}$

$$y_T = \frac{46,1 \cdot 12,0 + 32,2 \cdot 22,59}{46,1 + 32,2} = 16,36 \text{ m}$$

$$J = 4250 + 46,1 \cdot (16,36 - 12)^2 + 148 + 32,2 \cdot (22,59 - 16,36)^2 = 6524,12 \text{ cm}^4$$

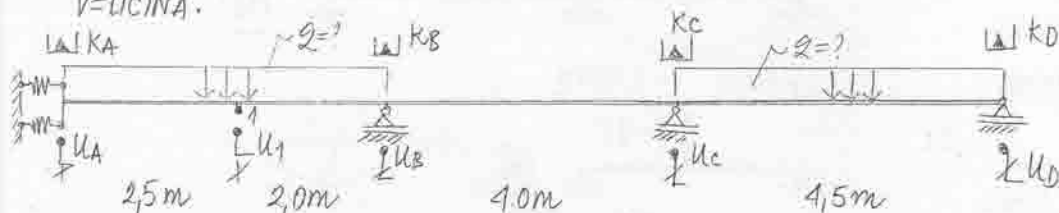
$$EJ = 21 \cdot 10^4 \cdot 6524,12 = 13700,65 \cdot 10^4 \text{ kNcm}^2 = 13700,65 \text{ kNm}^2$$

— AKO SHVATIMO DATI NOSAČ KAO GR=TRU=ELASTIČNO UKLO=ST=NU NA FIDNOM KRAJU, A N=POZNATI MOM=HT M KAO MOM=HT=ELASTIČNOG UKLO=ST=NA

$$M = -3 \frac{EJ}{l} \cdot \text{tg} \alpha_A + \left( \frac{3}{16} Pl \right) M_A^{\text{TOT}}$$

$$M = -\frac{13700,65}{6} \cdot 9,558 \cdot 10^{-4} + \frac{3}{16} \cdot 20 \cdot 6 = -654 + 22,5 \quad \boxed{M = 15,96 \text{ kNm}}$$

(\*) MAJ 2005. — ODR=ITI V=LIČINU PROBNOG OPT. „2“ HA OSNOVU IZMER=NIH D=FORMATIJSKIH V=LIČINA.



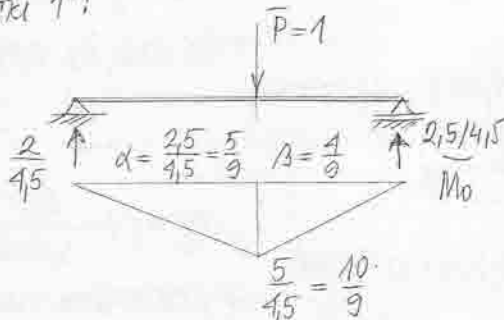
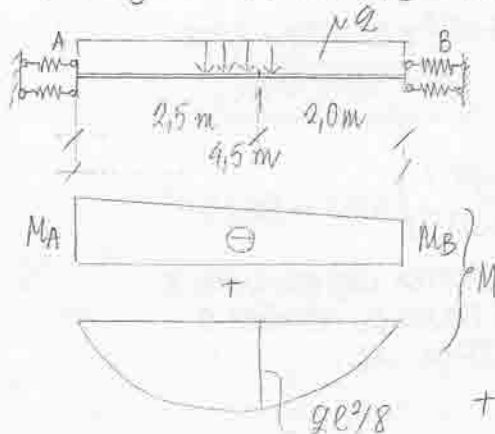
$E = 21 \cdot 10^4 \text{ kN/cm}^2$   
 $J = 2140 \text{ cm}^4$   
 $\mu = 0,01 \text{ mm}$

ST	KA	KB	KC	KD	KA	KB	KC	KD
0	0464	0156	0201	0205	0621	0+204	0+131	0+165
2	0464	0486	0201	0205	0621	0+048	1+130	2+011
ΔC	—	330	—	—	—	94	501	596
UT.	—	3,30	—	—	—	$4,831 \cdot 10^4$	$2,515 \cdot 10^3$	$3,063 \cdot 10^3$

$$\varphi'' = \Delta \alpha_K \cdot 1,06''$$

$$\varphi = \frac{\varphi''}{3600} \cdot \frac{\pi}{180}$$

ODR=DIVANJE IZRAZA ZA UGIB U TAČKI "1":



$$EJ \Delta y = \int M \bar{M} ds = -\frac{45}{6} \left[ M_A \left( 1 + \frac{4}{9} \right) + M_B \left( 1 + \frac{5}{9} \right) \right] \frac{10}{9} + \frac{45}{3} \frac{2,45^2}{8} \frac{10}{9} \left( 1 + \frac{5}{9} \right) = -\frac{5}{54} (13 M_A + 14 M_B) + \frac{505}{96} 2$$



$$= 3,2133 + 1,68452$$

$$= 1,68752 - 9,791$$

$$13M_A + 14M_B = 45,56252 - 95,3011$$

$$EJ\eta = -\frac{5}{54}(45,5625q - 95,3011) + \frac{505}{96}q = 1,0416q + 8,8242$$

$$1,04162 + 8,8242 = 2,12140 \cdot 33 \cdot 10^{-3} \rightarrow \boxed{q = 5,766 \text{ kN/m'}}$$

⑦ S=PT. I 2006. - NA OSNOVU REZULTATA MERENJA ODREDITI (NAKREDITI) DVA GRADU PRISUČNIH  
 814.  $\rho = 20 \text{ KN/m}^3$

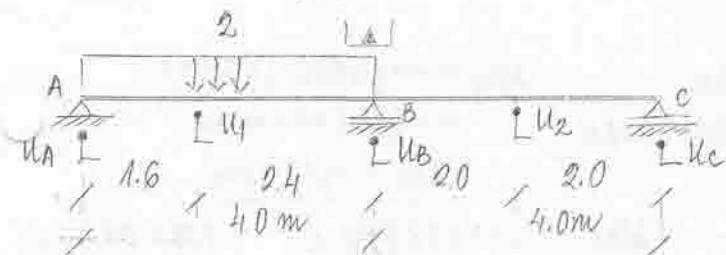
$$Q = 20 \text{ kN/m}$$

$$E = 2,1 \cdot 10^4 \text{ KN/cm}^2$$

$$J = 4344 \text{ cm}^4$$

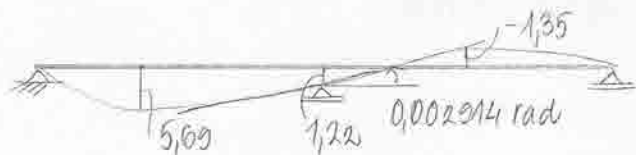
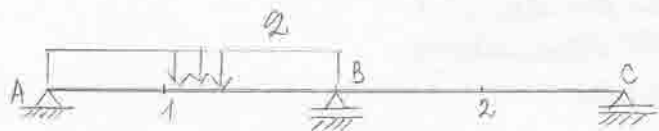
$$\lambda_m = 0,01 \text{ mm}$$

$$\rho_m = 1,06''$$



≡ LASTICNA LINIJA NOJAOA :

ST.	U <sub>A</sub>	U <sub>1</sub>	U <sub>B</sub>	U <sub>2</sub>	U <sub>C</sub>	K <sub>B</sub>
0	0247	0202	0306	0553	0404	04153
2	0247	0741	0428	0418	0404	74088
2-0	0	560	122	-135	0	567
UT	0	5,60	1,22	-1,35	0	60102"



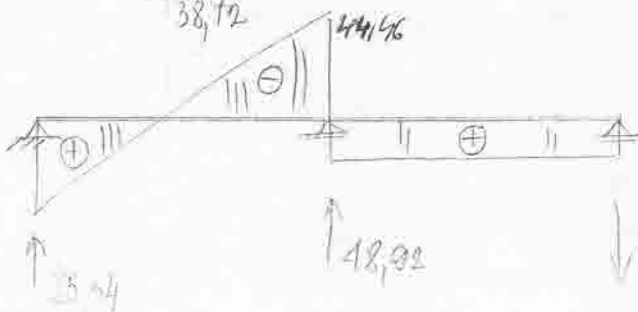
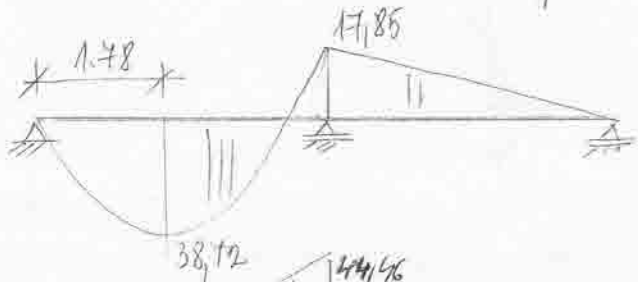
$$M_B = - \frac{3 \cdot 41 \cdot 4344}{4,0} (0,002914 - 0,000305) \quad M_B = - \frac{3EJ}{l} (\varphi_B - \varphi_{Bc})$$

$$M_B = -\frac{3EI}{l}(\psi_B - \psi_{BC})$$

$$M_B = -6849,8 \cdot 0,002609 = -17,85 \text{ kNm}$$

ODR = DIVANJ = MAX MU STAPN AB:  $M(x) = \frac{Q}{2}x(1-x) - 17,85 \cdot \frac{x}{9,0}$

$$\max M_{AB} = 10 \cdot 1,78(4 - 1,78) - 17,85 \frac{1,78^2}{4} = 38,72$$

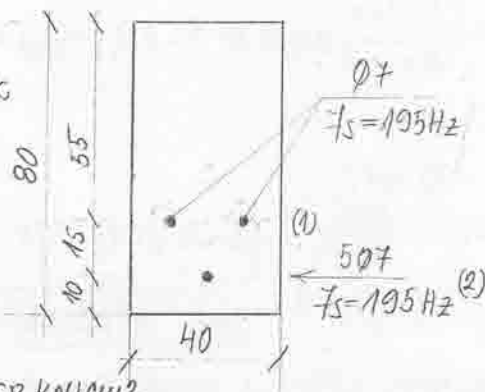
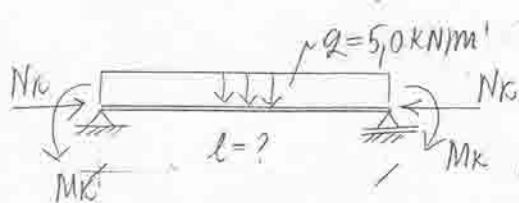


$$Q_{A-B} = -\frac{17,85}{4} + \frac{20,1}{2} = -4,4625 + 10 = 5,5375$$

$$Q_{BA} = -4.1625 - 40 = -44.1625$$

$$Q_{BC} = + \frac{17,85}{4} = + 4,4625$$

- ① D=C APS. 2004. → NAĆI RASPON  $\sigma_R = D \equiv$  U SL= D=ČA DVA SLUCAJA I UTVRDIŃ KW I JE UŠLOV M=RODAVAN. — A) AKO JE UKUPNI NAPON NA DONJOJ IVICI  $\sigma_d = 0$   
 B) AKO JE UKUPNI NAPON NA GORNJOJ IVICI  $\sigma_g = 0,5 \text{ kN/cm}^2$



— ODR=DIVANJE SIL= PR=PRODNOS  
 NAPR=ZANJA:  $\sigma_z = c \cdot l_i^2 \cdot f^2$

uzv.  $l_i = 100 \text{ cm}$

$$\sigma_{z1} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 195^2 = 121,68 \text{ kN/cm}^2$$

$$\sigma_{z2} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 195^2 = 121,68 \text{ kN/cm}^2$$

$$N_{k1} = A_{z1} \cdot \sigma_{z1} \rightarrow N_{k1} = 2,0385 \cdot 121,68 = 93,6936 \text{ kN}$$

$$A_{z1} = \frac{0,7^2 \pi}{4} = 0,385 \text{ cm}^2 \quad N_{k2} = 5,0385 \cdot 121,68 = 234,234 \text{ kN}$$

$$M_{k1} = -93,6936 \cdot (40 - 25) =$$

$$= -1405,404 \text{ kNcm}$$

$$M_{k1} = 14,05 \text{ kNm}$$

$$N_k = \sum N_{ki} = 93,6936 + 234,234 = 327,92 \text{ kN}$$

$$M_k = \sum M_{ki} = -14,05 - 70,27 = -84,32 \text{ kNm}$$

$$M_{k2} = -234,234 \cdot (40 - 10) = -7027,02 \text{ kNcm}$$

$$M_{k2} = -70,27 \text{ kNm}$$

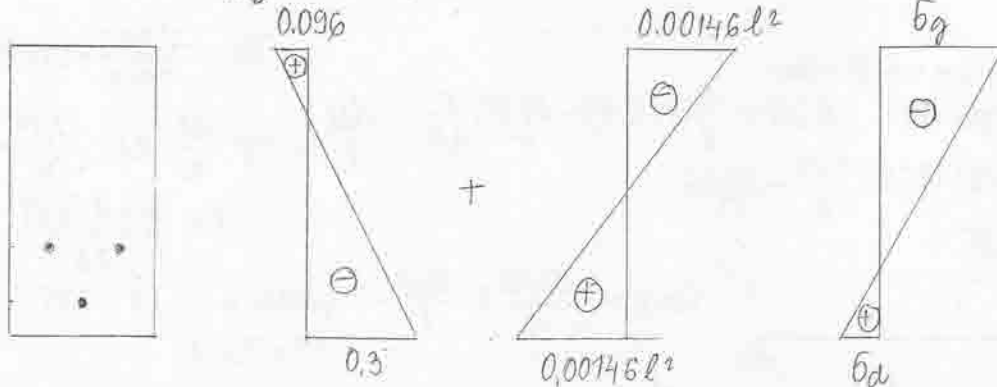
81A PRITISKA  
 ZAT=2= DONJO STRANU

— SRAČUNAVANJE NAPONA U PR=SEKUI U SR=EDINI RASPONA:

$$\sigma_{kd,g}^{\alpha-\alpha} = \frac{N_k}{A_b} \pm \frac{M}{W_b} = \frac{-327,92}{40 \cdot 80} \pm \frac{8432}{40 \cdot 80^2} \cdot 6 = -0,102 \pm 0,198$$

$$\sigma_{kd}^{\alpha-\alpha} = -0,3 \text{ kN/cm}^2 \quad \sigma_{kg}^{\alpha-\alpha} = 0,096$$

$$\sigma_{zg}^{\alpha-\alpha} = \pm \frac{M_{\alpha-\alpha}^2}{W_b} = \pm \frac{5,0 \cdot l^2}{8 \cdot 40 \cdot 80^2} \cdot 6 \cdot 100 = \pm 0,00146 l^2$$



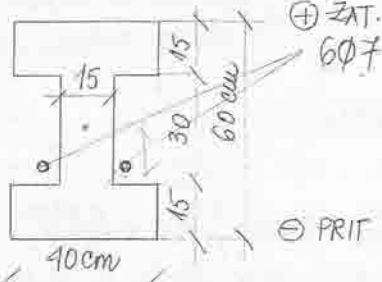
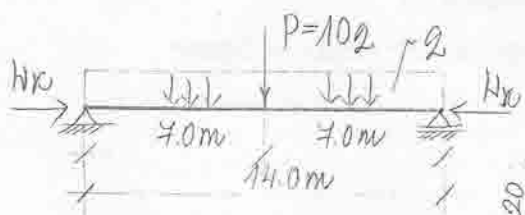
a) UŠLOV  $\sigma_d = 0$

$$\sigma_d = -0,3 + 0,00146 l^2 = 0 \rightarrow l = 14,33 \text{ m}$$

b) UŠLOV  $\sigma_g = -0,5 \text{ kN/cm}^2$

$$\sigma_g = 0,096 - 0,00146 l^2 = -0,5 \rightarrow l = 20,2 \text{ m}$$

- 2) U PRETHODNO NAPRUGNUTOM BET. NOSAČU IZVRŠNO JE MERENJE FREKVENCIE U KABLU ZA PRETHODNO NAPRUGANJE ( $f_{sr} = 200 \text{ Hz}$ ,  $l = 100 \text{ cm}$ ). SOPST. TEŽINA JE URAČUNATA U OPT. 2. NACI OPTREĆENJE P I 2 ZA SLEDUĆE SLUČAJEV: A) NAPON ZAT NA DONJOJ IVICI  $\sigma_{zd} = 0$ ; B) NAPON ZAT = ZANJA NA DONJOJ IVICI  $\sigma_{zd} = -0,056 \text{ pg}$ .



$$I_z^{(1)} = \frac{0,7^2 J}{4} = 0,385 \text{ cm}^2$$

$$A_k = 2 \cdot 6 \cdot 0,385 = 4,62 \text{ cm}^2$$

$$b_{ki} = 0,7^2 l^2$$

$$N_k = A_k \cdot b_{ki} = A_k \cdot 0,7^2 l^2$$

$$N_k = 4,62 \cdot 3,2 \cdot 10^{-7} \cdot 200^2 \cdot 100^2$$

$$N_k = 591,36 \text{ kN}$$

$$M_k = 591,36 \cdot 10 = 5913,6 \text{ kNm}$$

$$A_b = 2 \cdot 15 \cdot 40 + 30 \cdot 15 = 1650 \text{ cm}^2$$

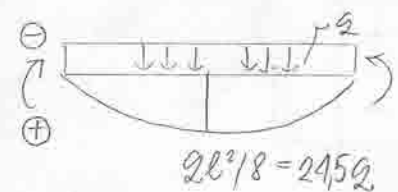
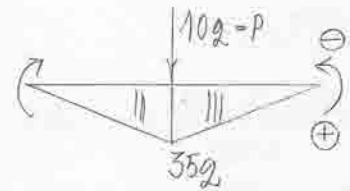
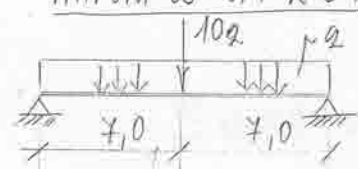
$$J_b = \frac{40 \cdot 60^3}{12} - 2 \cdot \frac{12,5 \cdot 30^3}{12} = 663750 \text{ cm}^4$$

$$W_b = \frac{663750}{30} = 22125 \text{ cm}^3$$

NAPONI OD PRETHODNO NAPRUGANJA:  $\sigma_b^{g,d} = -\frac{N_k}{A_b} \pm \frac{M_k}{W_b} = -\frac{591,36}{1650} \pm \frac{5913,6}{22125} = -0,3584 \pm 0,267$

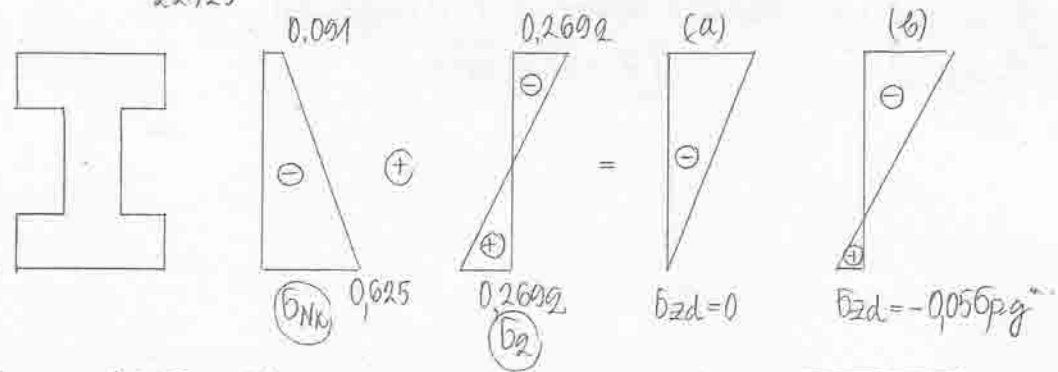
$$\sigma_b^g = -0,091 \text{ kN/cm}^2 \quad \sigma_b^d = -0,625 \text{ kN/cm}^2$$

NAPONI OD OPTREĆENJA:



$$M_{max} = 35q + 245q = 595q \text{ kNm} = 5950q \text{ kNm}$$

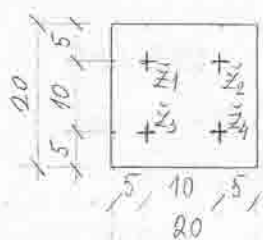
$$\sigma_b^{g,d} = \mp \frac{5950q}{22125} = 0,269q \text{ kN/cm}^2$$



(a)  $-0,625 + 0,269q = 0 \rightarrow q = 23,23 \text{ kN/m} \quad | \quad P = 10q = 232,3 \text{ kN}$

(b)  $-0,625 + 0,269q = -0,056 \cdot (-0,091 - 0,269q)$   
 $-0,625 + 0,269q = 0,00455 + 0,01345q$   
 $0,25555q = 0,62955 \rightarrow q = 24,65 \text{ kN/m} \quad | \quad P = 246,5 \text{ kN}$

- 3) ISPIITANA JE ZATEGA OD PRITISNO NAPREGNUTOG BETONA. ODREDITI KOLIKU SILU ZATEZANJA MOŽE PRIMATI ZATEGA POSLE MERENJA FREKVENCIJA SLOBODNOG OSCILOVANJA ŽICA ZA PRIDNAPREZANJE ( $l_{z1} = 100 \text{ cm}$ )



$$\phi_z = 7 \text{ mm}$$

$$f_{z1} = 95 \text{ Hz}$$

$$f_{z2} = 150 \text{ Hz}$$

$$f_{z3} = f_{z4} = 190 \text{ Hz}$$

$$f_{bz} = -0,1 \text{ KN/cm}^2$$

(DOZVOLJENI NAPON ZATEZANJA)

- SOPST. TEŽINA ZATEGE SE NE UZIMA U OBZIR

- SILA U POJEDINIM ŽICAMA:  $\phi_z = 7 \text{ mm}$ ;  $A_z = \frac{0,7^2 \pi}{4} = 0,385 \text{ cm}^2$

$$z_1: N_{z1} = A_z \cdot \sigma_z = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 95^2 = 11,12 \text{ kN}$$

$$z_2: N_{z2} = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 150^2 = 27,72 \text{ kN}$$

$$z_3, z_4: N_{z3} = N_{z4} = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 190^2 = 44,48 \text{ kN}$$

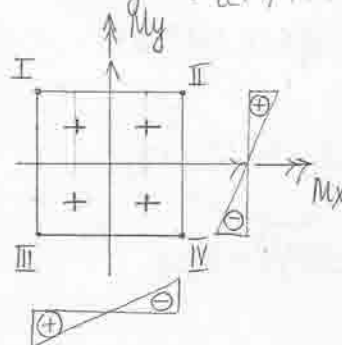
- UČINCI U ZATEZI OD PRIDNAPREZANJA:

$$N_k = \sum N_{ki} = 11,12 + 27,72 + 44,48 \cdot 2 = 127,8 \text{ kN}$$

$$M_{kx} = 2 \cdot 5 \cdot 44,48 - 5 \cdot (11,12 + 27,72)$$

$$M_{kxy} = 5 \cdot [(44,48 + 27,72) - (44,48 + 11,12)] = 83 \text{ kNm}$$

$$M_{kx} = 250,6 \text{ kNm}$$



$$A_b = 20^2 = 400 \text{ cm}^2$$

$$W_{b0} = \frac{20 \cdot 20^2}{6} = \frac{4000}{3} \text{ cm}^3$$

$$\sigma_b^I = \frac{127,8}{400} - \frac{250,6 + 83,0}{4000} \cdot 3 = 0,3195 - 0,2502 = 0,0693 \frac{\text{KN}}{\text{cm}^2}$$

$$\sigma_b^{IV} = \frac{127,8}{400} + \frac{250,6 + 83,0}{4000} \cdot 3 = 0,3195 + 0,2502 = 0,5697 \text{ KN/cm}^2$$

+ PRITISAK

$$\sigma_{bmin} = \sigma_{bmin}$$

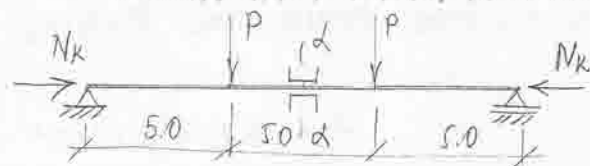
$$\sigma_{biv} = \sigma_{bmax}$$

$$\sigma_{bmin} - \frac{z}{A_b} = -0,1$$

$$z = A_b (0,1 + \sigma_{bmin}) = 400 (0,1 + 0,0693)$$

$$z = 67,72 \text{ kN}$$

- 4) ODR=DTI: - NORMALNU SILU PRETHODNOG NAPREZANJA POD USLOVOM DA JE PROSEČNA FREKV. SLOBODNOG OSCILOVANJA ŽICA U KABLOVIMA  $f=195\text{ Hz}$  NA DLEŽINI  $l=118\text{ cm}$ .  
- KORISNO OPT. P. PREMA MERENIM LOKALNIM DEFORMACIJAMA.



a) ODR=DNAN= SIL= PRETHODNOG NAPREZANJA:

$$a_{\bar{x}}^{(1)} = \frac{0,7^2 \sqrt{J}}{4} = 0,385 \text{ cm}^2$$

$$A_k = 2 \cdot 6 \cdot a_{\bar{x}}^{(1)} = 12 \cdot 0,385 = 4,62 \text{ cm}^2$$

$$\bar{\sigma}_x = c \cdot f^2 \cdot l^2 = 3,2 \cdot 10^{-7} \cdot 195^2 \cdot 118^2 = 169,43 \frac{\text{KN}}{\text{cm}^2}$$

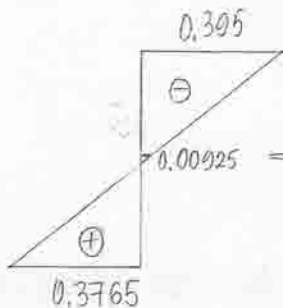
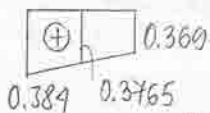
→ NAPON U ŽICI KABLA

$$N_k = A_k \cdot \bar{\sigma}_x = 4,62 \cdot 169,43 = 782,8 \text{ KN}$$

b)  $A_b = 60 \cdot 120 - 20 \cdot 80 = 5600 \text{ cm}^2$

$$J_b = \frac{1}{12} (60 \cdot 120^3 - 20 \cdot 80^3) = 7786666,667 \text{ cm}^4$$

$$W_b = \frac{J_b}{60} = 129777,8 \text{ cm}^3$$



OD SEKUNDARNOG UTICAJA

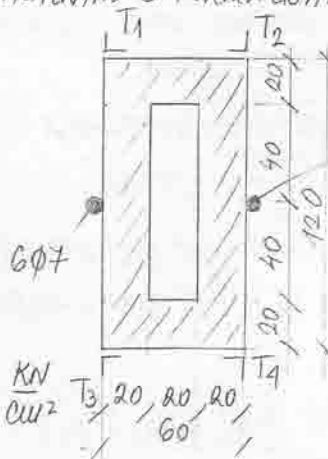
$$\sigma_{Mp} = \frac{M_p}{W_b} ; M_p = P \cdot 5,0 \text{ [KNm]}$$

OD MOMENTA SAVIJANJA (KOJI IZAZIVA SILA P)  
PRIMARNO OPT.

$$\sigma_{Mp} = \frac{5P}{W_b} \rightarrow P = \sigma_{Mp} W_b \cdot \frac{1}{500} = 0,386 \cdot 129777,8 / 500$$

↓ cm

$$P = 100,2 \text{ KN}$$



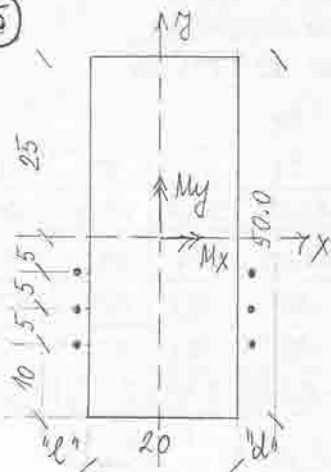
ST	T1	T2	T3	T4
0	16712	9637	6921	12688
P	16607	9534	7022	12785
$\Delta \bar{\sigma}$	-105	-103	101	97
$E \cdot 10^6$	-105	-103	101	97
$\bar{\sigma}$	-0.399	-0.391	0.384	0.369

$$E = \Delta \bar{\sigma} \cdot \rho \cdot \frac{K_i^{1,1}}{K_t} = \Delta \bar{\sigma} \cdot 10^{-6}$$

$$\bar{\sigma} = E_b \cdot E$$

$$E_b = 0,38 \cdot 10^4 \text{ KN/cm}^2$$

5



KOLIKI JE MOMENT NOSIVOSTI PR=DNAPR=GNUTOG BET. PRESKA  
UZ USLOV DA NE BUDU IZLOZENI ZATEZANJE. FR=KVENCJA PROBODNOG  
OSILOVANJA ZICA ZA PR=THOD. NAPREZANJE  $\sigma_{tlim} (f_t = 100 \text{ cm})$  IZNOSI  
U SVIM ZICAMA  $L=V0$   $f_t = 145 \text{ Hz}$ , A U SVIM ZICAMA ODSNO  $f_t = 155 \text{ Hz}$   
 $E_b = 0,35 \times 10^4 \text{ kN/cm}^2$

SILA PR=THODNOG NAPREZANJA:

$$\sigma_{t(1)} = \frac{0,7^2 \pi}{4} = 0,385 \text{ cm}^2$$

$$\sigma_{t(1)} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 145^2 = 67,28 \text{ kN/cm}^2$$

$$\sigma_{t(2)} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 155^2 = 76,88 \text{ kN/cm}^2$$

$$A_b = 20 \cdot 50 = 1000 \text{ cm}^2$$

$$W_{x-x} = \frac{20 \cdot 50^2}{6} = \frac{25000}{3}$$

$$W_{y-y} = \frac{20^2 \cdot 50}{6} = \frac{10000}{3}$$

$$N_{kl} = 3 \cdot 0,385 \cdot 67,28 = 77,71 \text{ kN}$$

$$N_{kd} = 3 \cdot 0,385 \cdot 76,88 = 88,80 \text{ kN}$$

$$N_k = N_{kl} + N_{kd} = 77,71 + 88,80 = 166,51 \text{ kN}$$

$$M_{kx} = N_k \cdot 10 = 166,51 \cdot 10 = 1665,1 \text{ kNcm}$$

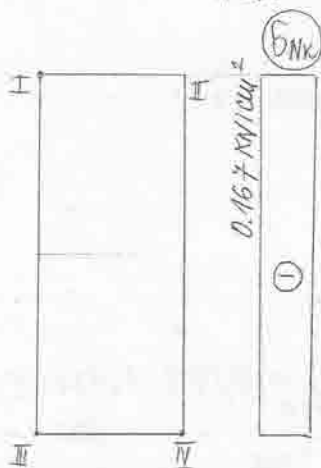
$$M_{ky} = 10 \cdot (N_{kd} - N_{kl}) = 10 \cdot (88,80 - 77,71) = 110,9 \text{ kNcm}$$

NAPONI U BETONSKOM PRESKU OD PR=THODNOG NAPREZANJA:

$$\sigma_{Nk} = - \frac{N_k}{A_b} = - \frac{166,51}{1000} = -0,167 \text{ kN/cm}^2$$

$$\sigma_{Mx}^k = \pm \frac{1665,1}{25000} \cdot 3 = \pm 0,2 \text{ kN/cm}^2$$

$$\sigma_{My}^k = \pm \frac{110,9}{10000} \cdot 3 = \pm 0,033 \text{ kN/cm}^2$$

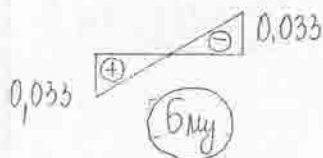


$$\sigma_I = 0,033 - 0,167 + 0,2 = 0,066 \rightarrow \text{MAX ZAT.}$$

$$\sigma_{II} = -0,033 - 0,167 + 0,2 = 0$$

$$\sigma_{III} = 0,033 - 0,167 - 0,2 = -0,334 \rightarrow \text{MAX T.}$$

$$\sigma_{IV} = -0,033 - 0,167 - 0,2 = -0,4$$

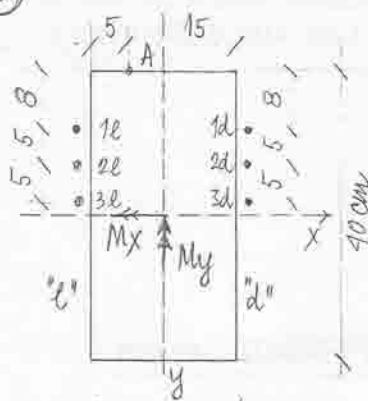


uvov zadatka:  $|\min \sigma_{b,pr}| = 0,334 = \max \sigma_{exp}$

$$\sigma = \frac{M_{HOS}}{25000} \rightarrow M_{HOS} = 27,83 \text{ kNm}$$



6



NA PRETHODNO NAPREGNUTOJ GREDI, POPREČNOJ PRESJEKA NA SKICI,  
IZMERE NE SU FRKVENCE SLOBODNOG OSCILOVANJA ŽICA ZA  
PRENAPRŽANJE  $\phi$  čelika ( $l = 100 \text{ cm}$ )

"e":  $f_1 = 130 \text{ Hz}$       "d":  $f_1 = 160 \text{ Hz}$   
 $f_2 = 135 \text{ Hz}$        $f_2 = 165 \text{ Hz}$   
 $f_3 = 139 \text{ Hz}$        $f_3 = 168 \text{ Hz}$

$$E_b = 0,35 \times 10^4 \text{ kN/cm}^2$$

ODREĐITI: A) SILE PRETHODNOG NAPRŽANJA  
 B) DILATACIJU I TACKU A

$$A_b = 20 \cdot 40 = 800 \text{ cm}^2$$

$$J_x = \frac{1}{12} 20 \cdot 40^3 = 106666,6 \text{ cm}^4$$

$$J_y = \frac{1}{12} 40 \cdot 20^3 = 26666,6 \text{ cm}^4$$

$$W_x^A = \frac{J_x}{20} = 5333,3 \text{ cm}^3$$

$$W_y^A = \frac{J_y}{10} = 5333,3 \text{ cm}^3$$

A)  $\alpha_i^{(1)} = \frac{0,7^2 \pi}{4} = 0,385 \text{ cm}^2$        $\bar{\alpha}_i = c \cdot f_i^2 \cdot l^2$        $N_{zi} = \alpha_i^{(1)} \cdot c \cdot f_i^2 \cdot l^2$

$$N_{z1}^e = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 130^2 \cdot 100^2 = 20,82 \text{ kN}$$

$$N_{z2}^e = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 135^2 \cdot 100^2 = 22,45 \text{ kN}$$

$$N_{z3}^e = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 139^2 \cdot 100^2 = 23,80 \text{ kN}$$

$$\sum N_k^e = 67,07 \text{ kN}$$

$$N_{z1}^d = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 160^2 \cdot 100^2 = 31,54 \text{ kN}$$

$$N_{z2}^d = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 165^2 \cdot 100^2 = 33,54 \text{ kN}$$

$$N_{z3}^d = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 168^2 \cdot 100^2 = 34,77 \text{ kN}$$

$$\sum N_k^d = 99,85 \text{ kN}$$

$$N_k = \sum N_k = 166,92 \text{ kN}$$

B)  $N = -N_k = -166,92 \text{ kN}$

$$E_b = 0,35 \cdot 10^4 \text{ kN/cm}^2$$

$$M_x = \sum_{i=1}^3 (N_i^e + N_i^d) \cdot e_{yi} \quad M_y = \sum_i (N_i^d - N_i^e) \cdot 10$$

$$M_x = (20,82 + 31,54) \cdot 12 + (22,45 + 33,54) \cdot 7 + (23,80 + 34,77) \cdot 2 = 1137,39 \text{ kN cm}$$

$$M_y = (99,85 - 67,07) \cdot 10 = 327,8 \text{ kN cm}$$

$$\bar{\sigma}_A = \frac{N}{A} - \frac{M_x}{W_x^A} + \frac{M_y}{W_y^A} = -\frac{166,92}{800} - \frac{1137,39}{5333,3} + \frac{327,8}{5333,3} = -0,3604 \text{ kN/cm}^2$$

$$\epsilon_A = \frac{\bar{\sigma}_A}{E_b} = -\frac{0,3604}{0,35 \cdot 10^4} = -103,0 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} = -0,103\% \quad \boxed{\epsilon_A = -0,103\%}$$

7

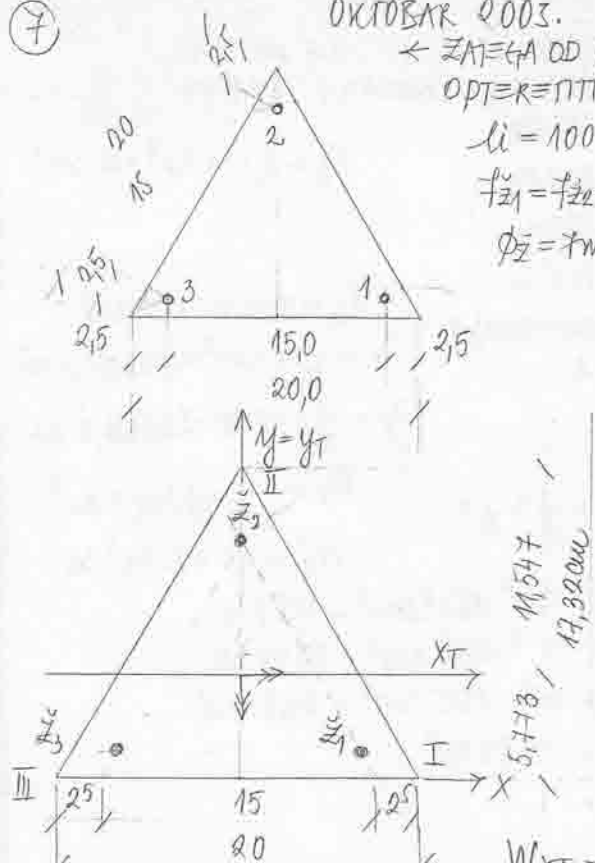
OKTOBAR 2003.

← ZAT=GA OD PR=DNAPR=GNUTOG B=TOHA. ODR=DTI KOJOM SILOM S=MOZ=OPT=K=ITI ZAT=GA UZ USLOV DA S= NE JAVI U NJOJ NAPON ZAT=ZANJA.

$$l_i = 100 \text{ cm}$$

$$f_{z1} = f_{z2} = 190 \text{ Hz} ; f_{z3} = 150 \text{ Hz}$$

$$\phi_z = 7 \text{ mm}$$



G=0m=TRISKE KARAKTERISTIKE B=TONSK=ZAT=GE:

$$A_z = \frac{1}{2} 20 \cdot 17.32 = 173.2 \text{ cm}^2$$

$$J_z^{xT} = \frac{1}{36} a l^3 = \frac{1}{36} 20 \cdot 17.32^3 = 2886.5 \text{ cm}^4$$

$$J_z^{yT} = 2 \cdot \frac{1}{12} a b^3 = 2 \cdot \frac{1}{12} 17.32 \cdot \left(\frac{20}{2}\right)^3 = 2886.7 \text{ cm}^4$$

$$W_{xI} = W_{xIII} = \frac{2886.5}{5.773} = 500 \text{ cm}^3$$

$$W_{xII} = \frac{2886.5}{11.547} = 249.978 \text{ cm}^3$$

$$W_{yI} = W_{yIII} = \frac{2886.7}{10} = 288.67 \text{ cm}^3 \quad W_{yII} = 0$$

KOORDINAT=TAČAKA:  $\check{z}_{1,2,3}$



$$x_{z1} = 7.5 \text{ cm}$$

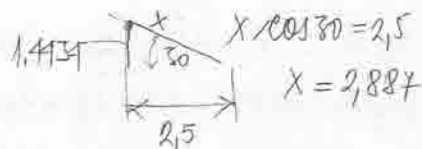
$$y_{z1} = -4.33 \text{ cm}$$

$$x_{z2} = 0$$

$$y_{z2} = 8.66 \text{ cm}$$

$$x_{z3} = -7.5 \text{ cm}$$

$$y_{z3} = -4.33 \text{ cm}$$



UTICAS PR=THODNODG NAPR=ZANJA:

$$A_z(i) = \frac{0.725}{4} = 0.385 \text{ cm}^2$$

$$\sigma_{z1} = \sigma_{z2} = 3.2 \cdot 10^{-7} \cdot 100^2 \cdot 190^2 = 115.52 \text{ KN/cm}^2$$

$$\sigma_{z3} = 3.2 \cdot 10^{-7} \cdot 100^2 \cdot 150^2 = 72 \text{ KN/cm}^2$$

$$N_{z1} = N_{z2} = A_z \cdot \sigma_z = 0.385 \cdot 115.52 = 44.475 \text{ KN} \quad N = \sum N_z = 116.67 \text{ KN}$$

$$N_{z3} = 0.385 \cdot 72 = 27.72 \text{ KN}$$

$$M_x = 44.475 \cdot (8.66 - 4.33 \text{ cm}) - 27.72 \cdot 4.33 = 72.55 \text{ KN cm}$$

$$M_y = -44.475 \cdot 7.5 + 27.72 \cdot 7.5 = -125.66 \text{ KN cm}$$

- NAPONSKO STANJE ZAT=GE OD PR=THODNODG NAPR=ZANJA:

$$\sigma_z^I = \frac{116.67}{173.2} + \frac{72.55}{500} - \frac{125.66}{288.67} = 0.383 \text{ KN/cm}^2$$

$$\sigma_z^{II} = \frac{116.67}{173.2} - \frac{72.55}{249.978} + 0 = 0.383$$

$$\sigma_z^{III} = \frac{116.67}{173.2} + \frac{72.55}{500} + \frac{125.66}{288.67} = 1.254$$

$$\sigma_{p, min} = 0.383 \text{ KN/cm}^2$$

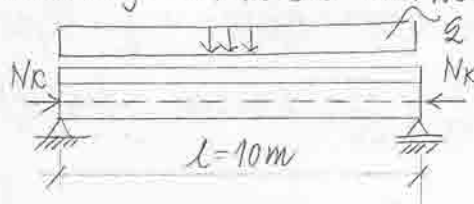
$$\frac{z}{173.2} = 0.383 \rightarrow z = 66.34 \text{ KN}$$

proveriti

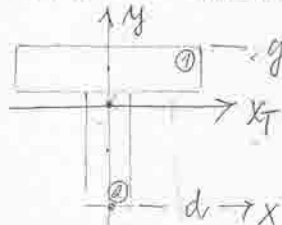
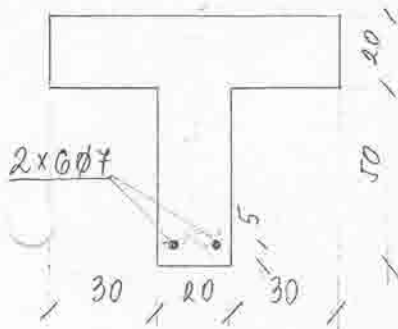
8) - NA KOM RASTOJANJU  $\lambda$  SE MOGU POSTAVITI NOSAČI KROVNOG POKRIVACA. PN JE OBAVLJENO NA STAZI ZA P.N I IZMERAENA JE PROSEČNA FREKVENCIJA OSCILOVANJA ŽICA KABLOVA ZA PN  $f = 205 \text{ Hz}$  NA DŽINI  $l = 80 \text{ cm}$ . NAGIB KROVNE RAVNI JE MALI ( $\cos \alpha \approx 1$ ,  $\sin \alpha \approx 0$ ), A PROJEKTO OPT. UZ SOPSIVNU TEŽINU JE  $\Delta q' = 1,20 \text{ kN/m}^2$ . RASTOJANJE  $\lambda$  IZMEĐU OVIH NOSAČA ODREDITI IZ MERODAVNOG OD TRI DATA USLOVA:

- $\sigma_{bp} \leq 1,6 \text{ kN/cm}^2$  (NAPON NA PRIT. IVICI)
- $\sigma_{bd} = 0$  (NEMA NAPONA ZAT. NA DONJOJ IVICI)
- $v \leq l/300$  (USLOV PO USIGUR ZA TOT. OPT)

$$q = g + \Delta q' \lambda$$



$G = 0$  METRUSKE KARAKTERISTIKE PRESKA:



$$\begin{aligned} A_1 &= 20 \cdot 80 = 1600 \text{ cm}^2 & y_1 &= 60 \text{ cm} \\ A_2 &= 20 \cdot 50 = 1000 \text{ cm}^2 & y_2 &= 25 \text{ cm} \\ A &= 2600 \text{ cm}^2 \\ y_{\text{IT}} &= \frac{1600 \cdot 60 + 1000 \cdot 25}{2600} = 46,5 \text{ cm} \end{aligned}$$

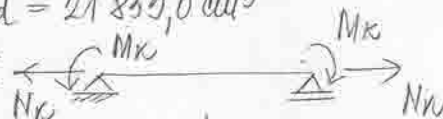
$$J = \frac{1}{12} (20^3 \cdot 80 + 20 \cdot 50^3) + 1600 (60 - 46,5)^2 + 1000 (46,5 - 25)^2$$

$$J = 1015516,6 \text{ cm}^4$$

$$y_g = 60 - 46,5 = 13,5 \text{ cm} \Rightarrow W_g = 43213,5 \text{ cm}^3$$

$$y_d = 46,5 \text{ cm} \rightarrow W_d = 21839,0 \text{ cm}^3$$

EF=KTI PRITHODNOG NAPREZANJA



$$f = 205 \text{ Hz} \quad l = 80 \text{ cm}$$

$$\sigma_x = c \cdot f^2 l^2 = 32 \cdot 10^{-7} \cdot 205^2 \cdot 80^2 = 80,067 \text{ kN/cm}^2$$

$$A_k = 12 \cdot \frac{0,7^2 \sqrt{5}}{4} = 4,62 \text{ cm}^2$$

$$N_k = A_k \cdot \sigma_x = 4,62 \cdot 80,067 = 397,63 \text{ kN}$$

$$e_k = 46,5 - 5 = 41,5 \text{ cm}$$

+ ZAT=ZANJ=

$$M_k = N_k \cdot e_k = 397,63 \cdot 41,5 = 16501,645 \text{ kNcm} = 165,02 \text{ kNm}$$

$$\sigma_{bg} = \frac{397,63}{2600} + \frac{165,02 \cdot 100}{43213,5} = 0,229 \text{ kN/cm}^2$$

$$E_{mn} = 0,3 \cdot 10^4 \text{ kN/cm}^2$$

$$\sigma_{bd} = \frac{397,63}{2600} - \frac{165,02 \cdot 100}{21839,0} = -0,909 \text{ kN/cm}^2$$

$\sigma = \text{DINIC} = ***$

$$f_k = - \frac{1}{8} \frac{M_k \cdot l^2}{EJ} = - \frac{1}{8} \frac{165,02 \cdot 10^2}{0,3 \cdot 1015516,6} = -0,007 \text{ m} = 7 \text{ mm} \leftarrow \text{NAGIB OD KABLOVA}$$

SPOL. OPTERECENJE:  $q = g + \Delta q' \lambda = 2600 \cdot 25 \cdot 10^{-4} + 1,20 \lambda = 6,5 + 1,2 \lambda$

$$1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

EF=KTI SPOL. OPT:

$$M = \frac{q l^2}{8} = \frac{100}{8} \cdot 2 = 12,52 \text{ kNm}$$

$$f_q = \frac{5}{384} \frac{q l^4}{EJ} = 0,000432 \text{ [m]}$$

$$\sigma_g^2 = - \frac{12,52 \cdot 100}{43213,5} = -0,029 \text{ kN/cm}^2$$

$$\sigma_d^2 = \frac{12,52 \cdot 100}{21839} = 0,057 \text{ kN/cm}^2$$

USLOVI ZADATAKA:

$$① \sigma_{bp} = \sigma_{bg} = 0,229 - 0,029q \geq -1,6 \quad q \leq \frac{1}{0,029}(1,6 + 0,229) = 63,07 \text{ KN/m'}$$

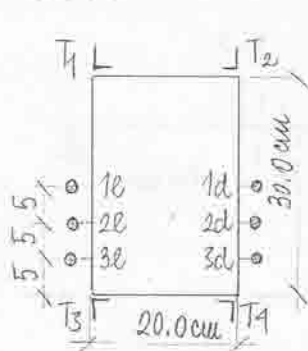
$$② \sigma_{bd} = 0 = -0,909 + 0,017q \rightarrow q = 15,95 \text{ KN/m'}$$

$$③ f_{d02} = \frac{l}{500} = \frac{10}{300} = 0,033 \text{ m} \quad \varphi = -0,007 - 0,00043q \leq 0,033$$
$$q \leq 93,8 \text{ KN/m'}$$

MERODAVNO ②  $q_{min} \rightarrow$

$$q = 15,95 = 6,5 + 1,2\lambda \rightarrow \boxed{\lambda \leq 7,875 \text{ m}}$$

9) Na pr=dnapr=ghutoj gr=di pr=opt=r=č=na izm=r=ne su frekvencije žice za pr=dnapr=zanj= (l=90cm). Po navedenim probnoj opt=r=č=na mer=ne su dilat=ci mer=im trak=ma. Nač= konačno naponsku stanj=. Rezultati mer=na dati su tab=lar=.



$$l = 90 \text{ cm}$$

$$\phi = 7 \text{ mm}$$

$$k_l = 2.15$$

$$k_t = 1.95$$

$$E_0 = 0.32 \times 10^4 \text{ kN/cm}^2$$

$$A_z^{(1)} = \frac{0.7^2 \pi}{4} = 0.385 \text{ cm}^2$$

f(Hz)	1	2	3
levo	126	133	138
deno	122	134	132

$$f_{sr} \quad 124 \quad 133.5 \quad 135$$

$$B_z = 0.7^2 l^2 \quad N_{ki} = 2 \cdot A_z^{(1)} \cdot c \cdot f_{sr}^2 l^2$$

$$N_{k1} = 2 \cdot 0.385 \cdot 3.2 \cdot 10^{-7} \cdot 124^2 \cdot 90^2 = 30.688 \text{ kN}$$

$$N_{k2} = 2 \cdot 0.385 \cdot 3.2 \cdot 10^{-7} \cdot 133.5^2 \cdot 90^2 = 35.570 \text{ kN}$$

$$N_{k3} = 2 \cdot 0.385 \cdot 3.2 \cdot 10^{-7} \cdot 135^2 \cdot 90^2 = 36.374 \text{ kN}$$

$$N_k = \sum N_{ki} = 102.633 \text{ kN}$$

$$M_k = 35.570 \cdot 5 + 36.374 \cdot 10 = 541.59$$

	T1	T2	T3	T4
D	18622	11208	13430	15720
P	18508	11099	13540	15828
ΔC	-114	-109	110	108
10 <sup>-6</sup> · E	-125692	-120119	121282	119074
B	-0.402	-0.385	0.388	0.381

$$E = \frac{k_l}{k_t} \cdot \Delta C = \frac{2.15}{1.95} \cdot 10^{-6} \cdot \Delta C$$

$$B = E \cdot C = 0.32 \cdot 10^4 \cdot E$$

$$A_0 = 20.30 = 600 \text{ cm}^2$$

$$W_0 = \frac{20.30^2}{6} = 3000 \text{ cm}^3$$

⊖ pritezan

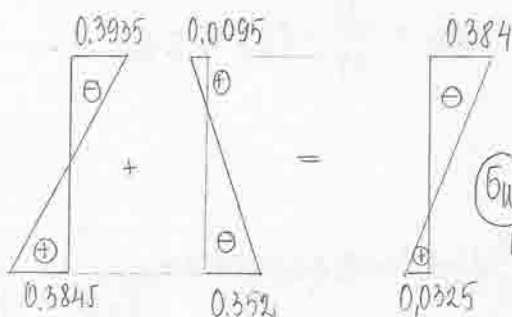
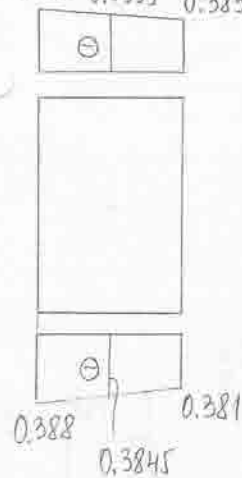
⊕ zat=zanj=

[kN/cm<sup>2</sup>]

$$\bar{\sigma}_k^{gornje} = - \frac{102.633}{600} + \frac{541.59}{3000} = 0.0095 \text{ kN/cm}^2$$

$$\bar{\sigma}_k^{dole} = - \frac{102.633}{600} - \frac{541.59}{3000} = -0.352 \text{ kN/cm}^2$$

$$0.402 \quad 0.385 \quad 0.385$$



⊖ [kN/cm<sup>2</sup>]

ukupno napr=zanj=

$$0.3845$$

$$0.352$$

$$0.3825$$

$$\downarrow$$

$$\downarrow$$

od probnog  
opt=r=č=na

od pr=dnapr=zanj=

10. Na pravoj strani napregnutoj bet. gredi pre = eksploatacionog opt. izmeren su frekv. zica pri oscilovanju na duzini  $l_0 = 90$  cm. Odrediti opt. p. uz uslov da na p. na donjoj ivici  $p = s_{ka}$  bud. = 0. Uzeti u obzir slob. t. z. i nu nosaca.

$$A_b = 40.70 - 20.50 = 1800 \text{ cm}^2 = 0.18 \text{ m}^2 \quad f_b = 25 \text{ kN/m}^3$$

KABLOVI

$$\sigma_{xi} = c \cdot f_i^2 \cdot l^2 \quad l_0 = 90 \text{ cm}$$

$$N_{ki} = A_{ki} \cdot b_{ki}$$

$$A_{ki} = \frac{0,725}{4} = 0,181$$

SILA PR=TRONOG NAPR=2AN,A :  $N_k = \sum N_k = 821,58 \text{ kN}$

$MOMENT (U ODNOSU NA T. PR=SKA) \equiv KSCENTRICNOSTI \quad M_K = 143,84 \cdot 0,25 + 149,48 \cdot 0,125$   
 $+ 134,72 \cdot 0,25 + 134,72 \cdot 0,125 = 105,165 \text{ KNM}$   
 novsi.  $B_{NK} = \frac{M_K}{N_K} = - \frac{821,58}{-4564,33} \text{ KN} = 0,18 \text{ KN}$

UNTUK:  $\sigma_{BK} = \frac{N_k}{A_b} = -\frac{821,58}{0,18} = -4564,33 \frac{\text{KN}}{\text{m}^2} = -0,456 \text{ KN/cm}^2 + 134,7$

$$\sigma_{br}^{gld} = \pm \frac{M_{ro}}{W_b} = \pm \frac{105,165 \cdot 100}{26714,3} = \pm 0,394 \text{ kN/cm}^2$$

$$J_b = \frac{1}{12} (40.70^3 - 20.50^3) = 935\,000 \text{ cm}^4 \quad W_b = \frac{J_b}{35} = 26\,714,3 \text{ cm}^3$$

$$\sigma_{bg}^k = -0,456 + 0,394 = -0,062 \text{ kN/cm}^2$$

$$b_{bd}^k = -0,456 - 0,394 = -0,85 \text{ KN/cm}^2$$

- Иницијал од својо опфрџа:

$$M_g = \frac{1}{8} 4,5 \cdot 15^2 = 126,5625 \text{ KNm}$$

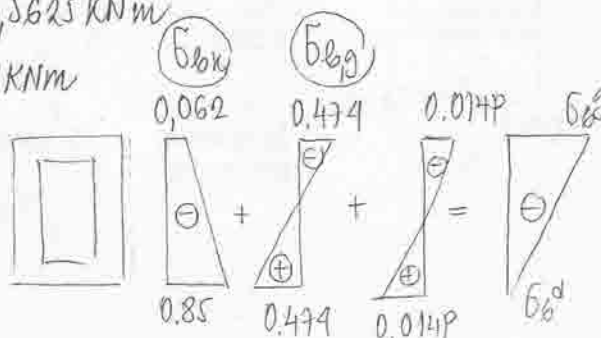
$$M_p = \frac{1}{4} P \cdot 15 = 3,75 P \text{ KNm}$$

$$56,8^{gid} = + \frac{126,5625 \cdot 100}{26714,3} = + 0,474 \text{ KN/cm}^2 \text{ KN/cm}^2$$

$$\sigma_{p, \text{mid}} = \pm \frac{3,75 \text{ P} \cdot 100}{26714,3} = \pm 0,014 \text{ P kN/cm}^2$$

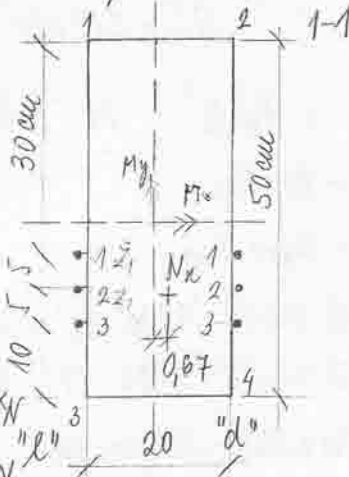
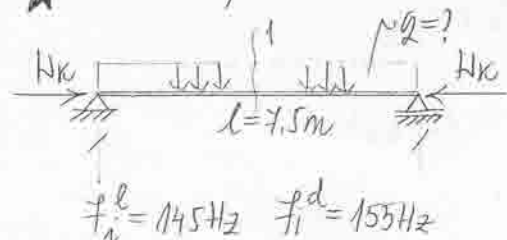
Условие:  $\sigma_{bd} = 0$

$$-0,85 + 0,474 + 0,014P = 0 \rightarrow P = 26,8 \text{ kN}$$





- 11)  $D = 2003$ . ODR = DITI  $V = \text{LIČINA KORISNOG OPT. UZ USLOV DA SE NA PRITIVICI NE JAVI VEĆI NAPON OD 12 MPa, NITI NA ZATIVICI VEĆI OD 0,5 MPa.}$   $\phi_{\text{sum}} = 100 \text{ cm}$



$G = 0 \text{ m. KARAKT. PR = S = KA}$   
 $A_b = 20 \cdot 30 = 1000 \text{ cm}^2$   
 $I_{bx} = \frac{20 \cdot 30^3}{12} = \frac{625000}{3}$   
 $W_{bx} = \frac{25000}{3} \quad i = 1,2,3$   
 $I_{by} = \frac{20^3 \cdot 30}{12} = \frac{100000}{3}$   
 $W_{by} = \frac{10000}{3} \quad i = 1,2,3$

$1 \text{ MPa} = 10^{-1} \frac{\text{KN}}{\text{cm}^2}$

UTICAJI PR = THODNOG NAPREZANJA:

$A_z = \frac{0,7^2 J}{4} = 0,385 \text{ cm}^2$

$N_{ki}^l = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 145^2 = 25,903 \text{ KN}$

$N_{ki}^d = 0,385 \cdot 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 155^2 = 29,599 \text{ KN}$

$N_k^l = 3 \cdot 25,903 = 77,709 \text{ KN}$

$N_{kd} = 88,797 \text{ KN} \rightarrow N_k = 166,506 \text{ KN}$

$y_k = \frac{(25,903 + 29,599) \cdot 5 + 55,502 \cdot 10 + 55,502 \cdot 15}{166,506} = 10 \text{ cm}$

$x_k = \frac{10 \cdot 3 \cdot (29,599 - 25,903)}{166,506} = 0,67 \text{ cm}$

NAPONSKO STANJE OD PN:  $N_k = -166,506 \text{ KN}$   $M_{kx} = -166,506 \cdot 10 = -1665,06 \text{ KNm}$

$M_{ky} = 166,506 \cdot 0,67 = 111,559 \text{ KNm}$

$\sigma_{b1k} = -\frac{166,506}{1000} + \frac{1665,06}{25000} \cdot 3 + \frac{111,559}{10000} \cdot 3 = -0,167 + 0,200 + 0,033 = 0,066 \text{ KN/cm}^2$

$\sigma_{b2k} = -0,167 + 0,200 - 0,033 = 0 \text{ KN/cm}^2$

$\sigma_{b3k} = -0,167 - 0,200 + 0,033 = -0,334 \text{ KN/cm}^2$

$\sigma_{b4k} = -0,167 - 0,200 - 0,033 = -0,4 \text{ KN/cm}^2$

NAPONSKO STANJE OD OPT = R = C = N, A:

$\sigma_{b1,2q} = -\frac{2 \cdot 7,5^2}{8} \cdot 100 \cdot \frac{3}{25000} = -0,0842 \text{ (KN/cm}^2\text{)}$

$\sigma_{b3,4q} = +0,0842$

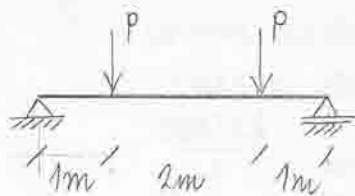
ODR = DIVANJE OPT = R = C = N, A:  $\rightarrow N = \text{MAX OPT. NA PRITIVICI OD } N_k \text{ u KN/m}$

① uslov  $\rightarrow -1,2 = -0,0842 \rightarrow \boxed{q = 14,29 \text{ KN/m}}$

② uslov  $\rightarrow 0,5 \frac{1}{10} = -0,334 + 0,0842 \Rightarrow \boxed{q = 4,57 \text{ KN/m}}$   $\leftarrow \text{MERODAVNO OVO MANJE}$

- 12)  $P = ?$  TAKO DA NAPON NA DONJOJ IVICI  $\sigma_{\sigma z} = 0$ . U ZICAMA KABLA ZA PN MERENAJE  $f_{K1} = f_{K2} = 195 \text{ Hz}$  SLOBODNOS OSCILOVANJA NA MZINI  $\lambda = 75 \text{ cm}$ .

$$f_{K1}^{ST} = 210 \text{ Hz} \quad f_{K2}^{ST} = 195 \text{ Hz}$$



$$\sigma_{K1} = c \cdot f_{K1}^2 \cdot l_i^2 \quad A_K = G \cdot \frac{0,5^2 \pi}{4} = 1,18 \text{ cm}^2$$

$$N_{K1} = A_K \cdot \sigma_{K1}$$

$$N_K = N_{K1} + N_{K2}$$

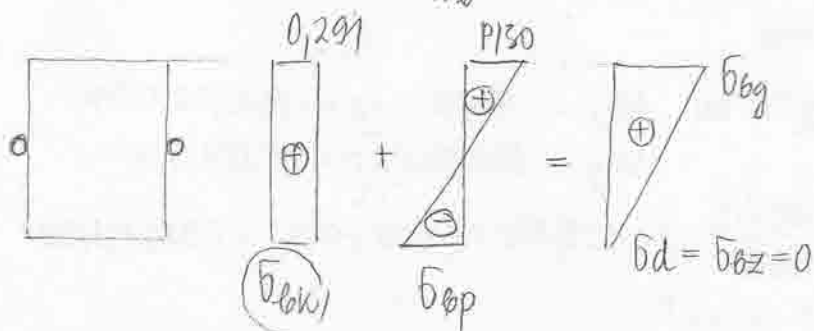
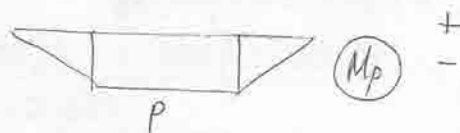
$$N_{K1} = 1,18 \cdot 3,2 \cdot 10^{-7} \cdot 210^2 \cdot 75^2 = 93,67 \text{ kN}$$

$$N_{K2} = 1,18 \cdot 3,2 \cdot 10^{-7} \cdot 195^2 \cdot 75^2 = 80,76 \text{ kN}$$

$$N_K = N_{K1} + N_{K2} = 174,44 \text{ kN}$$

$$\sigma_{\sigma K} = \frac{174,44}{20 \cdot 30} = 0,291 \text{ kN/cm}^2$$

$$\sigma_{\sigma P} = \pm \frac{M_P}{W_b} = \pm \frac{P \cdot 100}{20 \cdot 30^2} \cdot 6 = \pm \frac{P}{30}$$



$$0,291 + \frac{P}{30} = 0 \rightarrow P = 30 \cdot 0,291 = \underline{\underline{8,73 \text{ kN}}}$$

- 13) U MOHTAZHOM „SISTEMU IMS“ V=2A IZMENDU SIMBOVA I TAVANICA SE OSTVARUJE TRENING. TRENING IZAZIVAJU SILE PN KABLOVA (607), KOJI PROLAZE U POSEBNOJ FAZI SLOBODNOG KROZ KANAL U TAVANICAMA. KONTROLA SILE JE VRŠENA MERENJEM FREKVENCIJE SLOBODNOG OSCILOVANJA ŽICA KABLA NA BAZI  $L=100\text{ cm}$ . MERENJE JE VRŠENO NA 2 VRSTE KABLOVA (RAZLIČITA DULJINA) PRE I POSLE „UKLINJAVANJA“. KOLIKO SU OSTVARENE SILE I GUBICI SILE OD PROKUZIVANJA KUNA? OBJASNI!

KABL	PR.	DULJINA	FR (Hz) PREUKL	FR POSLE UKLINJ.
1	607	25m	142 Hz	130 Hz
2	607	5m	142 Hz	123 Hz

REZULTATI MERENJA:  $A_k = 6 \frac{0,7^2 \pi}{4} = 2,37 \text{ cm}^2$

1) KABL ( $L=25,0\text{ m}$ )  
 $f_0 = 142 \text{ Hz}$   $\bar{b}_{k0} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 142^2 = 64,52 \text{ kN/cm}^2$   
 $N_{k0} = 64,52 \cdot 2,37 = 149,05 \text{ kN}$

- POSLE UKLINJAVANJA:

$f_1 = 130 \text{ Hz}$   $\bar{b}_{k1} = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 130^2 = 61,83 \text{ kN/cm}^2$   
 $N_{k1} = 2,37 \cdot 61,83 = 142,82 \text{ kN}$   
 G SILA U KABLU POSLE UKLINJAVANJA

$\Delta N_k = 149,05 - 142,82 = 6,23 \text{ kN}$  (GUBITAK SILE OD PROKUZIVANJA)

$\epsilon_{k1} = \frac{61,83}{2,1 \cdot 10^4} = 2944,3 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \rightarrow \Delta l_{k1} = 2944,3 \cdot 10^{-6} \cdot 25000 = 73,61 \text{ mm}$

$\Delta k = 76,8 - 73,61 = 3,2 \text{ mm}$  (UVLAČENJE KUNA, TO IZAZIVA GUBITAK SILE)

2) KABL ( $L=5,0\text{ m}$ )

$f_0 = 142 \text{ Hz}$   $\bar{b}_{k0} = 64,52 \text{ kN/cm}^2$   $\epsilon_{k0} = 3072,4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \rightarrow \Delta l_{k0} = 15,4 \text{ mm}$   
 $N_{k0} = 149,05 \text{ kN}$

POSLE UKLINJAVANJA:

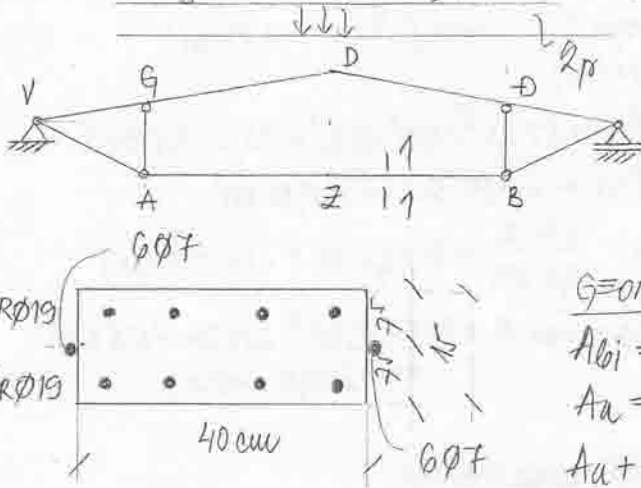
$f_1 = 123 \text{ Hz} \rightarrow \bar{b}_{k1} = 48,41 \text{ kN/cm}^2 \rightarrow N_{k1} = 11,83 \text{ kN}$

GUBITAK SILE:  $\Delta N_k = 37,22 \text{ kN}$ ;  $\epsilon_{k1} = \frac{48,41}{2,1 \cdot 10^4} = 2305 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \rightarrow \Delta l_{k1} = 11,53 \text{ mm}$

$\Delta k = 15,4 - 11,53 = 3,87 \text{ mm}$

Uključivanje ( $\Delta k$ ) približno isto, jer su iste početne sile pritiska na uklin  $N_{k0}$ . Kod kraćeg kabla su veći efekti u gubitku sile, jer su isti silu kraći kabl treba manje izdužiti ( $\frac{76,8}{15,4} \approx 5$  puta)

- 14) VRSNO JE ISPITIVANJE AB NOSAČA SA ZAT=40M, POSLE SANACIJE ZAT=9 (D=0 A-B). SANACIJA JE IZVRŠENA UNOSIENJEM SILA PRITISKA U D=0 A-B POMOĆU ŽICA ZA PREDNAPREŽANJE, KOJE SU ANKEROVANE U CVOROVIMA A I B, SILU PRITISKA PRI SANACIJI MERIMO FREKVENCIJOM SLOBODNOG OSCILOVANJA ŽICA NA  $\lambda = 100$  CM. DATI, RAČUNSKU ARGUMENTOVAN ODGOVOR, ZA KOLIKO SE PROMENI FREKVENCIJA SLOBODNOG OSCILOVANJA ŽICA OČAKUJE POVEĆANJE SILA ZAT=ZAN, A U ZAT=21 (D=0 A-B) OD IZOBNOG OPT.  $\Delta Z_{p.0} = 200$  KN.



- PROSČENA VREDNOST SLOBODNOG OSCILOVANJA ŽICA OČ:  $f_z = 160$  Hz
- SILA U ZAT=21 OD SIT. NOSAČA JE  $Z_g = 200$  KN
- NOSIVOST BETONA NA ZAT=ZAN:

$$b_z = 30 \text{ MPa}$$

G=0M=TRUSKE KARAKTERISTIKE AB ZAT=9:

$$A_{bi} = 40 \cdot 15 + (6-1) \cdot 8 \cdot 2,84 = 713,6 \text{ cm}^2 \leftarrow n = \frac{E_c}{E_s} = \frac{21}{9,35} = 6$$

$$A_a = 2 \cdot 4 \cdot 2,84 = 22,72 \text{ cm}^2$$

$$A_i = A_b + (n-1) A_c$$

$$A_a + A_{cp} = 22,72 + 2 \cdot 6 \cdot \frac{0,7^2 \cdot 5}{4} = 22,72 + 12 \cdot 0,385 = 27,34 \text{ cm}^2$$

I FAZA: NOSAČ OPT. SAMO PODST T=ZINOM  $\rightarrow Z_g = 200$  KN  $\leftarrow$  SILA U ZAT=21

$$b_{bz} = \frac{Z_g}{A_{bi}} = \frac{200}{713,6} = 0,280 \frac{\text{KN}}{\text{cm}^2} < 0,3 \rightarrow \text{BETON NEĆE ISPRSKATI}$$

II FAZA: VRSI SE SANACIJA ZAT=9 A SE PREDNAPREŽE SILOM (PRITISKA)

$$b_z = c \cdot l^2 \cdot f^2 = 3,2 \cdot 10^{-4} \cdot 100^2 \cdot 160^2 = 81,92 \text{ KN/cm}^2$$

$$Z_{pN} = -N_k = -81,92 \cdot 12 \cdot 0,385 = -378,47 \text{ KN} \rightarrow \text{ZAT=9A Ć BITI PRINJENUTA}$$

$$\text{SILA U ZAT=21: } Z = Z_g + Z_{pN} = 200 - 378,47 = -178,47 \text{ KN}$$

$$\text{NAPON U BETONU: } b_{bII} = 0,280 - \frac{378,47}{713,6} = 0,28 - 0,53 = -0,25 \text{ KN/cm}^2$$

$$Z_{III} = Z_g + Z_{pN} + Z_p = +200 - 378,47 + 200 = +21,53 > 0$$

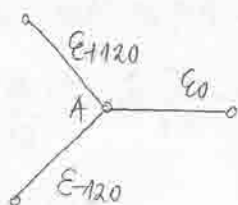
$$\text{NAPON U BETONU: } b_{bIII} = b_{bII} + \Delta b_{III} = -0,25 + \frac{200}{713,6} = -0,25 + 0,28 = 0,03 \text{ KN/cm}^2$$

> 0

< 0,3 KN/cm<sup>2</sup>

NE MA PRSILNA, NIJE DOŠLO DO DEKOMPR=SIJE ZAT=9,  
NE MENJA SE SILA U KABLOVIMA  $f = \text{const.}$

- ① MART 2007. → NA ČEĆIČNOM ELEMENITU JE PUTEM ROZJE, PREMA SKICI MERENO DEFORMACIONSKO STANJE U TAČKI A. RACUNSKIM I GRAFIČKIM POSTUPKOM ODREDITI:



$$\epsilon_0 = +480 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{+120} = +17 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{-120} = +305 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

A) DEFORMACIONSKO I NAPONSKO STANJE U TAČKI A

B) DILATACIJU KOJA JE POD UGLOM OD 90° U ODNOSU NA 0°-PRAVAC

C) NACRTATI ORIJENTACIJU KRSTA GLAVNIH NAPONA U TAČKI A, AKO JE 0° PRAVAC HORIZONTALNI PRAVAC.

$$\epsilon_{+120} = 17 \cdot 10^{-6}$$



$$\epsilon_0 = 480 \cdot 10^{-6}$$

$$\epsilon_{-120} = 305 \cdot 10^{-6}$$

$$\epsilon_{+60} = 305 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_0 = 480 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{-60} = 17 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

IRACUNSKI POSTUPAK

$$a) \epsilon_{1,2} = \frac{\epsilon_0 + \epsilon_{60} + \epsilon_{-60}}{3} \pm \frac{1}{3} \sqrt{(2\epsilon_0 - \epsilon_{60} - \epsilon_{-60})^2 + 3(\epsilon_{60} - \epsilon_{-60})^2}$$

$$10^6 \epsilon_{1,2} = \frac{1}{3}(480 + 305 + 17) \pm \frac{1}{3} \sqrt{(2 \cdot 480 - 17 - 305)^2 + 3 \cdot (17 - 305)^2}$$

$$= 267,3 \pm \frac{1}{3} \sqrt{638^2 + 3 \cdot (-288)^2} = 267,3 \pm 269,95 \rightarrow \epsilon_1 = 537,28 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\rightarrow \epsilon_2 = -2,62 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \left| \frac{\epsilon_{-60} - \epsilon_{60}}{2\epsilon_0 - \epsilon_{60} - \epsilon_{-60}} \right| \cdot \sqrt{3} = \left| \frac{-288}{638} \right| \cdot \sqrt{3} = 0,78187 \rightarrow \alpha_0^* = 19,01^\circ$$

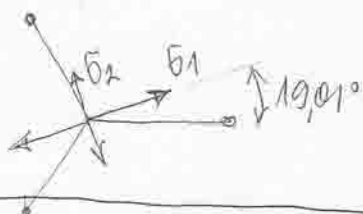
$$B < 0 \quad 170 \quad \alpha_0 = \alpha_0^* \quad \leftarrow$$

$$\alpha_0 = \alpha_0^* = 19,01^\circ \quad \leftarrow$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{21 \cdot 10^9}{1-0,3^2} (537,28 - 0,3 \cdot 2,62) \cdot 10^{-6} = 12,38 \text{ kN/cm}^2$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{21 \cdot 10^9}{1-0,3^2} (-2,62 + 0,3 \cdot 537,28) \cdot 10^{-6} = 3,66 \text{ kN/cm}^2$$

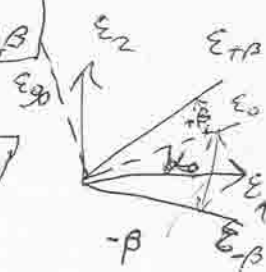
$$b) \epsilon_{90^\circ} = \frac{\epsilon_1 + \epsilon_2}{2} - \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\alpha_0 = 267,33 \cdot 10^{-6} - 269,95 \cdot 10^{-6} \cos 2 \cdot 19,01^\circ = 54,66 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$



$$\epsilon_0 = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\alpha_0$$

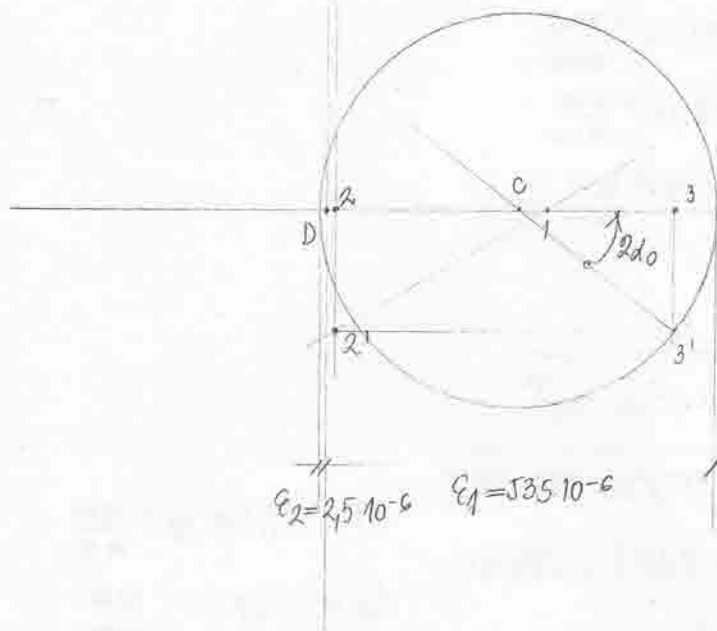
$$\epsilon_\beta = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\alpha_0 \cos 2\beta - \frac{\epsilon_1 - \epsilon_2}{2} \sin 2\alpha_0 \sin 2\beta$$

$$\epsilon_{-\beta} = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\alpha_0 \cos 2\beta + \frac{\epsilon_1 - \epsilon_2}{2} \sin 2\alpha_0 \sin 2\beta$$



## II GRAFICKI POSTUPAK

1812



$$c = \frac{\epsilon_0 + \epsilon_{60} + \epsilon_{-60}}{3} = \frac{802 \cdot 10^{-6}}{3} = 267,3$$

$$\gamma_{\epsilon} = 100 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \approx 1 \frac{\text{mm}}{\text{mm}}$$

$$\gamma_{\epsilon} = \frac{E}{1+\nu} \gamma_{\epsilon} = \frac{21 \cdot 10^9}{1+0,3} \cdot 100 \cdot 10^{-6}$$

$$= 1,615 \frac{\text{KN}}{\text{mm}^2} \approx 1 \frac{\text{mm}}{\text{mm}}$$

$$\frac{\bar{C}N}{\bar{C}D} = \frac{1+\nu}{1-\nu} = \frac{1,3}{0,7}$$

$$\bar{C}N = 1,857 \bar{C}D$$

$$\bar{C}N = 1,857 \cdot 2,65 = 4,92 \text{ mm}$$

$\gamma_{\epsilon}$



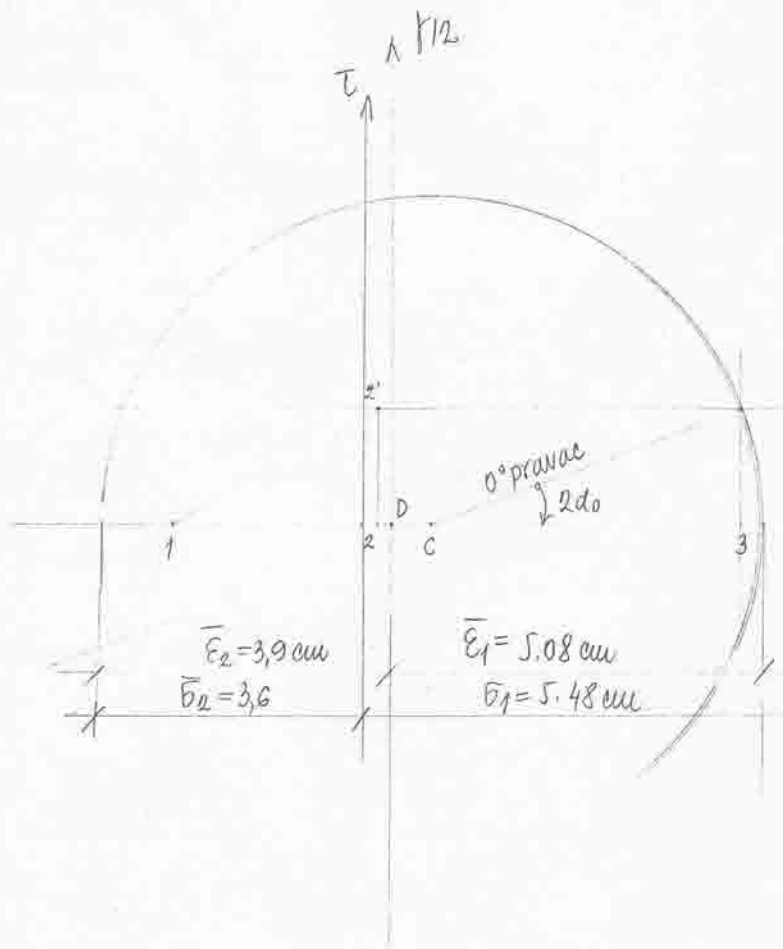
$$\epsilon_0 = 480 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad (3)$$

$$\epsilon_{+120} = -17 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} = \epsilon_{-60} \quad (2)$$

$$\epsilon_{-120} = -305 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} = \epsilon_{+60} \quad (1)$$

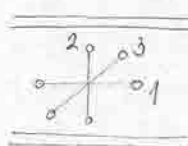
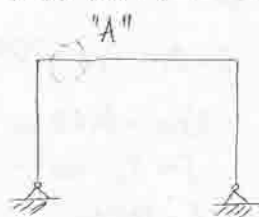
$$c = 52,6$$

$$\gamma_d = 1 \text{ cm} \approx 100 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$



$\gamma \epsilon$

- 2) NA JEDNOM ČLIČNOM ELEMENTU PUTEM ROZETI MERENE SU LOKALNE DEFORMACIJE, PRI FAZNOM OPTEREĆENJU DEFORMETROM PFENDER, PODACI MERENJA SU DANI TABELARNO. NAČI DEFORMACIJSKO I NAPONSKO STANJE ZA OBA OPTEREĆENJA, A ZATIM NAČI KVANTITATIVNU I KVANTITATIVNU RAZUKU IZMEĐU FAZA OPT.  $l_{pf} = 100 \text{ mm}$



$\beta = 45^\circ$



STANJE	M1	M2	M3	M4
0	0462	0328	0193	0611
P <sub>1</sub>	0409	0345	0172	0607
P <sub>2</sub>	0433	0367	0236	0609
P <sub>1</sub> -0	-53	17	-21	-4
P <sub>2</sub> -0	-29	39	43	-2
$\Delta_1 - \Delta K_1$	-49	21	-17	
$\Delta_1 - \Delta K_2$	-27	41	45	
$\epsilon_1 \cdot 10^{-6}$	-490	210	-170	
$\epsilon_2 \cdot 10^{-6}$	-270	410	450	

$$l = 100 \text{ mm} \rightarrow p_{pf} = 10 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_1 = (\Delta_1 - \Delta K_1) \cdot p_{pf}$$

$$\epsilon_2 = (\Delta_1 - \Delta K_2) \cdot p_{pf}$$

$$\epsilon_0 = \epsilon_{3-3} \quad \epsilon_{+45} = \epsilon_{2-2} \quad \epsilon_{-45} = \epsilon_{1-1}$$

STANJE 1:  $\epsilon_{+45} = 210 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_0 = -170 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_{-45} = -490 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$

$$\epsilon_{1,2} = \frac{\epsilon_{45} + \epsilon_{-45}}{2} \pm \frac{1}{2} \sqrt{(2\epsilon_0 - \epsilon_{45} - \epsilon_{-45})^2 + (\epsilon_{-45} - \epsilon_{45})^2}$$

$$\epsilon_{1,2} \cdot 10^6 = \frac{210 - 490}{2} \pm \frac{1}{2} \sqrt{(-2 \cdot 170 - 210 + 490)^2 + (-490 - 210)^2}$$

$$\epsilon_{1,2} \cdot 10^6 = -140 \pm \frac{1}{2} \sqrt{(-60)^2 + (-700)^2} = -140 \pm 351,283 \quad \epsilon_1 = 211,283 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_2 = -491,283 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \left| \frac{\epsilon_{-45} - \epsilon_{45}}{2\epsilon_0 - \epsilon_{45} - \epsilon_{-45}} \right| = \left| \frac{-700}{-60} \right| = 11,667 \rightarrow \alpha_0^* = 42,55^\circ \quad \begin{matrix} B < 0 \\ I < 0 \end{matrix} \quad \alpha_0 = 90 - \alpha_0^* \leftarrow$$

$$\alpha_0 = 90 - 42,55 = 47,45^\circ$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{21 \cdot 10^4}{1-0,3^2} (211,283 - 0,3 \cdot 491,283) \cdot 10^{-6} = 1,47 \text{ kN/cm}^2$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{21 \cdot 10^4}{1-0,3^2} (-491,283 + 0,3 \cdot 211,283) \cdot 10^{-6} = -0,87 \text{ kN/cm}^2$$

STANJE 2:  $\epsilon_{+45} = 410 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_0 = 450 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_{-45} = -270 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$

$$\epsilon_{1,2} \cdot 10^6 = \frac{410 - 270}{2} \pm \frac{1}{2} \sqrt{(2 \cdot 450 - 410 + 270)^2 + (-270 - 410)^2}$$

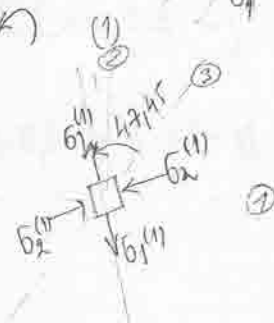
$$\epsilon_{1,2} \cdot 10^6 = 70 \pm \frac{1}{2} \sqrt{(760)^2 + (-680)^2} = 70 \pm 509,902$$

$$\epsilon_1 = 579,902 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_2 = -439,902 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

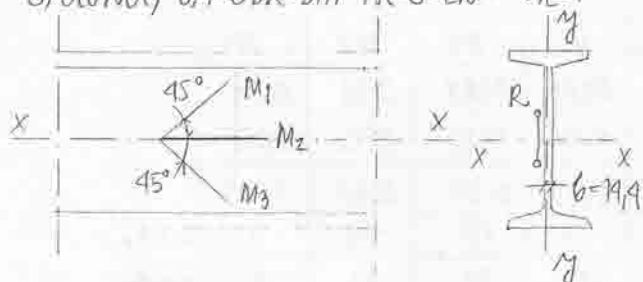
$$\tan 2\alpha_0^* = \left| \frac{-680}{760} \right| = 0,8947 \rightarrow \alpha_0^* = 20,91^\circ \rightarrow \begin{matrix} B < 0 \\ I > 0 \end{matrix} \quad \alpha_0 = \alpha_0^* \leftarrow$$

$$\sigma_1 = \frac{21 \cdot 10^4}{1-0,3^2} (579,902 - 0,3 \cdot 439,902) \cdot 10^{-6} = 10,34 \text{ kN/cm}^2$$

$$\sigma_2 = \frac{21 \cdot 10^4}{1-0,3^2} (-439,902 + 0,3 \cdot 579,902) \cdot 10^{-6} = -6,14 \text{ kN/cm}^2$$



- 3)  $S = P 04$ ,  
 NA KONSTRUKTIVNOM ELEMENTU OD ČELIKA (KOJI NIJE OPTEREĆEN NA TORZIJU) ROZETOM  
 (MERENIM TRAKAMA) SU MERENE DILATKJE U 3 PRAVCA, PRAVAC  $M_2$  JE POKLAPA NA ROZETI  
 SA TEŽIŠNOM OSMOM NOSAČA (X OSA), U PRAVCU "Y" (UPRAVNO NA  $M_2$ ) NEMA NIKAKVOG  
 SPOLNOG OPT. ODRŽITI PRESJECNI  $S = 857$



$$K_t = K_t \quad E = 2,1 \cdot 10^4 \text{ kN/cm}^2 \quad \nu = 0,3$$

	$M_1$	$M_2$	$M_3$
O	12352	10641	08329
P	12113	10498	08468
P-O	-239	-143	139
E	-239	-143	139

$$I 400: J_{x-x} = 29210 \text{ cm}^4$$

$$W_{x-x} = 1460 \text{ cm}^3$$

$$S = 857 \text{ cm}^2$$

$$A = 118 \text{ cm}^2$$

$$\epsilon_0 = \epsilon_{M_2} = -143 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{45} = \epsilon_{M_1} = -239 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{135} = \epsilon_{M_3} = 139 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon = \frac{K_t}{K_t} \Delta \sigma \cdot 10^{-6} \rightarrow \text{PODATAK MEREN TRAK}$$

$$\epsilon_{1,2} = \frac{\epsilon_{45} + \epsilon_{135}}{2} \pm \frac{1}{2} \sqrt{(\epsilon_0 - \epsilon_{45} - \epsilon_{135})^2 + (\epsilon_{45} - \epsilon_{135})^2}$$

$$\epsilon_{1,2} \cdot 10^6 = \frac{-239 + 139}{2} \pm \frac{1}{2} \sqrt{(-2 \cdot 143 + 239 - 139)^2 + (139 + 239)^2}$$

$$\epsilon_{1,2} \cdot 10^6 = -50 \pm \frac{1}{2} \sqrt{(-186)^2 + (378)^2} = -50 \pm 210,64$$

$$\epsilon_1 = 160,64 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_2 = -260,64 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \frac{378}{-186} = -2,032$$

$$\rightarrow \alpha_0^* = 31,9^\circ \quad \text{BROJ } 1 < 0 \rightarrow \alpha_0 = 90 - \alpha_0^* \rightarrow$$

$$\alpha_0 = 58,1^\circ \rightarrow \text{OD } 0^\circ \text{ PRAVCA}$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{2,1 \cdot 10^4}{1-0,3^2} (160,64 - 0,3 \cdot 260,64) \cdot 10^{-6} = 1,903 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{2,1 \cdot 10^4}{1-0,3^2} (-260,64 + 0,3 \cdot 160,64) \cdot 10^{-6} = -4,903 \frac{\text{kN}}{\text{cm}^2}$$

NAPONSKO STANJE U PRESJECNU:

USLOVI ZADATKA:  $\sigma_y = 0$  U PRAVCU Y NEMA NIKAKVOG SPOLNOG OPT.

$$\tau = \tau_0 \quad (\tau = 0)$$

$$\sigma_x = \sigma_N \quad (\sigma_M = 0)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

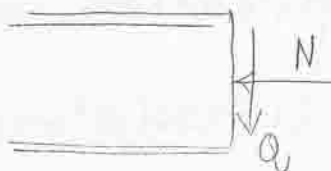
$$\sigma_1 + \sigma_2 = \sigma_x = \sigma_N \quad \sigma_N = 1,903 - 4,903 = -3,00 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_1 - \sigma_2 = 2 \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} \rightarrow \sqrt{\left(\frac{3}{2}\right)^2 + \tau^2} = \frac{1,903 + 4,903}{2} = 3,403 \frac{\text{kN}}{\text{cm}^2}$$

$$\rightarrow \tau = 3,055 \frac{\text{kN}}{\text{cm}^2}$$

$$Q = \frac{\tau \cdot b \cdot J}{S} = \frac{3,055 \cdot 144 \cdot 29210}{857} = 149,94 \text{ kN}$$

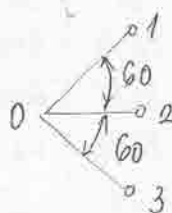
$$N = \sigma_N \cdot A = -3,0 \cdot 118 = -354 \text{ kN}$$



4

AB = ELEMENT, M = KREĆE IZVRŠENO INSTRUMENTOM PFENDER SA MAX VELOCITOM BAZE  
POMERENIJA INSTRUMENTA ODRĘDITÓF. I NAPÓNÓKÓ STANJE U DATÓJ TAČKÓ MÉTÓDÓM  
PO ÍZBÓRÓ (ANALÓCKÍ ÍLÍ GRÁFÍCKÍ)

STANJE	0-1	0-2	0-3	"K"
0	0637	0428	0718	0279
OPT	0636	0410	0710	0281
0	0632	0427	0718	0279
$\Delta_1$	5	-18	-8	2
$\Delta_2$	4	-17	-8	2
$\Delta_{SR}$	4,5	-17,5	-8	2
$\Delta_{ST-AX}$	2,5	-19,5	-10	—
$\epsilon \cdot 10^6$	25	-195	-100	—



$$E_0 = 0,32 \cdot 10^9 \text{ KN/cm}^2$$

$$\nu_0 = 0,15$$

$$\rho_{pf} = 10 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \text{za } l_{max} = 100 \text{ mm}$$

$$(+ZAT = ZAH =)$$

$$\times 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{0-1} = \epsilon_{+60} = 25 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_{0-2} = \epsilon_0 = -195 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_{0-3} = \epsilon_{-60} = -100 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{1,2} = \frac{1}{3} (\epsilon_{60} + \epsilon_0 + \epsilon_{-60}) \pm \frac{1}{3} \sqrt{(2\epsilon_0 - \epsilon_{60} - \epsilon_{-60})^2 + 3(\epsilon_{-60} - \epsilon_{60})^2}$$

$$10^6 \cdot \epsilon_{1,2} = \frac{1}{3} (25 - 195 - 100) \pm \frac{1}{3} \sqrt{(-2 \cdot 195 - 25 + 100)^2 + 3(-100 - 25)^2}$$

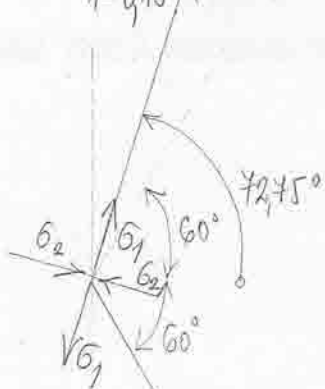
$$= -90 \pm \frac{1}{3} \sqrt{(-315)^2 + 3(-125)^2} = -90 \pm 127,41$$

$$\epsilon_1 = 37,41 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \epsilon_2 = 217,41 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

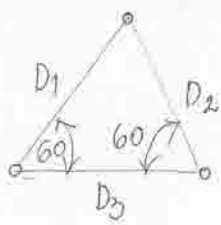
$$\tan 2\alpha_0^* = \left| \frac{-125}{-315} \right| \sqrt{3} = 0,68732 \rightarrow \alpha_0^* = 17,25^\circ \quad \begin{matrix} B < 0 \\ 1 < 0 \end{matrix} \quad \alpha_0 = 90^\circ - \alpha_0^* = 72,75^\circ$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{0,32 \cdot 10^9}{1-0,15^2} (37,41 - 0,15 \cdot 217,41) \cdot 10^{-6} = 0,016 \text{ KN/cm}^2$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{0,32 \cdot 10^9}{1-0,15^2} (-217,41 + 0,15 \cdot 37,41) \cdot 10^{-6} = -0,693 \text{ KN/cm}^2$$



- 5) MAJ 2002. → AB ELEMENT, DEFORMACIJE MERENE LABISKONOM ( $l = 250 \text{ mm}$ ). RACUNSKI I GRAFIČKI ODREDITI DEFORMACIONSKI I NAPONNO STANJE.



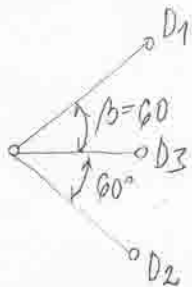
$$E_b = 0.30 \times 10^9 \text{ kN/cm}^2$$

$$\nu_b = 0.18$$

STANJE	D1	D2	D3	D4
0	5850	6190	5910	5560
P	5875	6215	5935	5570
0	5870	6210	5930	5580
$\Delta_1$	25	25	25	10
$\Delta_2$	5	5	5	-10
$\Delta_{sr}$	15	15	15	0
$\Delta_{sr Ak}$	15	15	15	—
$\epsilon \cdot 10^6$	60	60	60	✓

$$\rho = 4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

+ PRITISAK



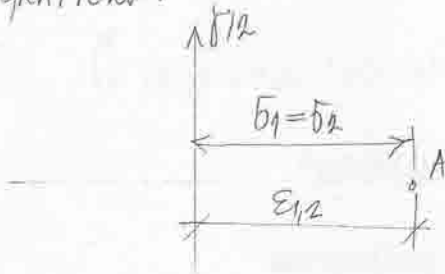
$$\epsilon_0 = \epsilon_{D3} = \epsilon_{160} = \epsilon_{D1} = \epsilon_{60} = \epsilon_{D2} = 60 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{1,2} = 60 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha^* = 0 \text{ Neodredeno IZRAZ}$$

$$\sigma_1 = \sigma_2 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{E \epsilon_1}{1-\nu} = \frac{0.30 \cdot 60 \cdot 10^{-6} \cdot 10^9}{1-0.18} = 0.220 \text{ kN/cm}^2 \text{ (PRITISAK)}$$

TAČKA U OVOJ OKOLINI JE MERENO JE IZOTROPNA GRAFIČKI:



$$\tau_D = 1 \text{ cm} = 20 \cdot 10^{-6}$$

$$\tau_u = \frac{E}{1-\nu} \tau_d = \frac{0.3 \cdot 10^9 \cdot 20 \cdot 10^{-6}}{1-0.18}$$

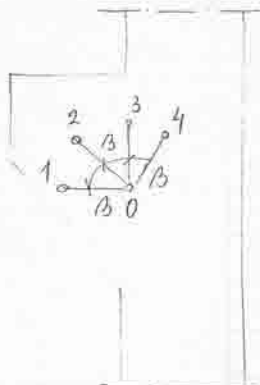
$$\tau_u = 1 \text{ cm} = 0.073 \text{ kN/cm}^2$$

$$\bar{\sigma}_A = 3 \text{ cm} = 0.219 \text{ kN/cm}^2$$

$$\Delta = 0.5\%$$

U OVOM SLUCAJU GREŠKA JE BEZHATAJNA, ZAVISNO OD RAZMERE, IZNOSI NEKOLIKO % I PROISTICE IZ ZAKRUGLJIVANJA

- 6) NA ELEMENTU BET KONSTRUKCIJE MERENJE SU DILATACIJE PUTEM ROZETA U 4 PRAVCA, NAČI GLAVNE DILATACIJE GRAFIČKIM I RAČUNSKIM POSTUPKOM



$$E_b = 0,358 \cdot 10^9 \text{ KN/cm}^2$$

$$\epsilon_{1-1} = -14 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

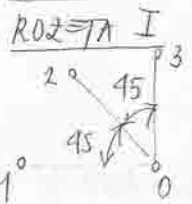
$$\epsilon_{3-3} = 4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{2-2} = -38 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{4-4} = 30 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\nu = 0,20$$

#### A) RAČUNSKI POSTUPAK



$$\epsilon_0 = \epsilon_{2-2} = -38 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{1,2} = \frac{\epsilon_{45} + \epsilon_{135}}{2} \pm \frac{1}{2} \sqrt{(\epsilon_0 - \epsilon_{45} - \epsilon_{135})^2 + (\epsilon_{45} - \epsilon_{135})^2}$$

$$\epsilon_{145} = \epsilon_{1-1} = -14 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$10^6 \epsilon_{1,2} = \frac{-14 + 4}{2} \pm \frac{1}{2} \sqrt{(-2 \cdot 38 + 14 - 4)^2 + (4 + 14)^2}$$

$$\epsilon_{-45} = \epsilon_{3-3} = 4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$10^6 \epsilon_{1,2} = -5 \pm \frac{1}{2} \sqrt{(-66)^2 + (18)^2} \rightarrow$$

$$\tan 2\alpha_0^* = \left| \frac{18}{-66} \right| < 0 \rightarrow \alpha_0^* = 7,6276^\circ$$

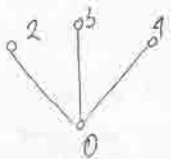
$$\epsilon_1 = 29,205 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}; \epsilon_2 = -39,205 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\rightarrow \alpha_0 = 90 - \alpha_0^* \rightarrow \alpha_0 = 82,37^\circ$$

$$\sigma_1 = \frac{E_b}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{0,358 \cdot 10^4}{1-0,2^2} (29,205 - 0,2 \cdot 39,205) \cdot 10^{-6} = 0,08 \text{ KN/cm}^2$$

$$\sigma_2 = \frac{E_b}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{0,358 \cdot 10^4}{1-0,2^2} (-39,205 + 0,2 \cdot 29,205) \cdot 10^{-6} = -0,12 \text{ KN/cm}^2$$

#### ROZETA II



$$\epsilon_0 = 4 \cdot 10^{-6}$$

$$\epsilon_{145} = -38 \cdot 10^{-6}$$

$$\epsilon_{-45} = 30 \cdot 10^{-6}$$

$$\epsilon_2 = 10 \cdot 10^{-6} = 1 \text{ cm} = r_2$$

$$\epsilon_{1,2} = \frac{30 - 38}{2} \pm \frac{1}{2} \sqrt{(2 \cdot 4 - 30 + 38)^2 + (-38 - 30)^2}$$

$$10^6 \epsilon_{1,2} = -4 \pm \frac{1}{2} \sqrt{(16)^2 + (-68)^2} = -4 \pm 34,928$$

$$\epsilon_1 = 30,928 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_2 = -38,928 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \left| \frac{-68}{16} \right| < 0 = 4,25 \rightarrow \alpha_0^* = 38,38^\circ$$

$$r_2 = \frac{E_b}{1+\nu} r_1 = 0,03 \frac{\text{KN}}{\text{cm}^2} \quad \alpha_0 = \alpha_0^* \leftarrow$$

$$1^\circ C = \frac{\epsilon_{45} + \epsilon_{135}}{2} = -5 \cdot 10^{-6}$$

$$2^\circ (1) \rightarrow \epsilon_{45}$$

$$2\alpha_0 = 82,37^\circ \cdot 2 = 165^\circ$$

$$76,5^\circ$$

$$\frac{CN}{CD} = \frac{1+\nu}{1-\nu} = \frac{1,2}{0,8} = 1,5 \quad CN = 1,5 \cdot CD = 1,5 \cdot 0,5 = 0,75 \text{ cm}$$

$$\epsilon_2 = 39 \cdot 10^{-6}$$

$$\epsilon_1 = 29 \cdot 10^{-6}$$

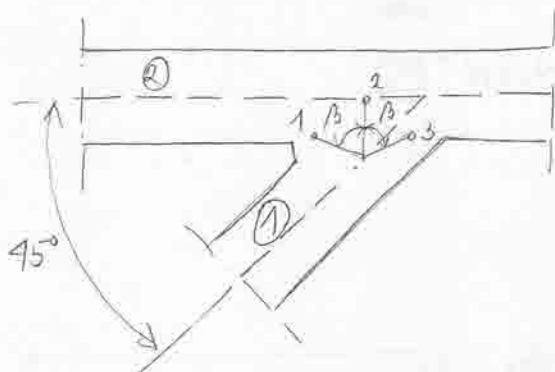


7) U ČVORU ČELIČNOG ELEMENTA, NA SJAJM 1, MEREÑO JE DEFORM. STANJE POKREĆU ROZET. ZA MEREÑO DILATACIJA KORISĆENE SU MERNE TRAKE  $k_1 = k_2$ . ODREDITI RAČUNSKI I GRAFIČKI DEFORMACIJSKO I NAPONSKO STANJE, PA NA OSNOVU NJEGA OCENITI KARAKTER NAPREŽAJNA STAPA 1.

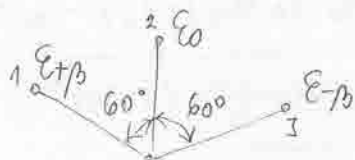
$$\beta = 60^\circ \quad E_c = 2,1 \times 10^4 \text{ kN/cm}^2$$

$$\nu = 0,30$$

$$p_T = 1,0 \times 10^{-6} \frac{\text{mm}}{\text{mm}}$$



STANJE	0-1	0-2	0-3
O	18210	13490	09521
P	18130	13538	09928
O	18214	13408	09517
$\Delta_1$	-80	128	407
$\Delta_2$	-84	130	411
$A_{SR}$	-82	129	409
$10^6 \epsilon$	-82	129	409



$$\epsilon_0 = 129 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{+60} = -82 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{-60} = 409 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

RAČUNSKI POSTUPAK:  $\epsilon_{1,2} = \frac{\epsilon_{60} + \epsilon_0 + \epsilon_{-60}}{3} \pm \frac{1}{3} \sqrt{(2\epsilon_0 - \epsilon_{60} - \epsilon_{-60})^2 + 3(\epsilon_{-60} - \epsilon_{60})^2}$

$$10^6 \epsilon_{1,2} = \frac{-82 + 129 + 409}{3} \pm \frac{1}{3} \sqrt{(2 \cdot 129 + 82 - 409)^2 + 3 \cdot (409 + 82)^2}$$

$$= 152 \pm \frac{1}{3} \sqrt{(-69)^2 + 3 \cdot (491)^2} \Rightarrow \epsilon_1 = 436,41 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_2 = -132,41 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \left| \frac{491}{-69} \right| \sqrt{3} \Rightarrow \alpha_0^* = 42,681^\circ$$

$$\rightarrow \alpha_0 = 90^\circ - \alpha_0^* = 47,319^\circ \searrow$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = 9,15 \text{ kN/cm}^2 \quad \sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = 0,0343 \text{ kN/cm}^2$$

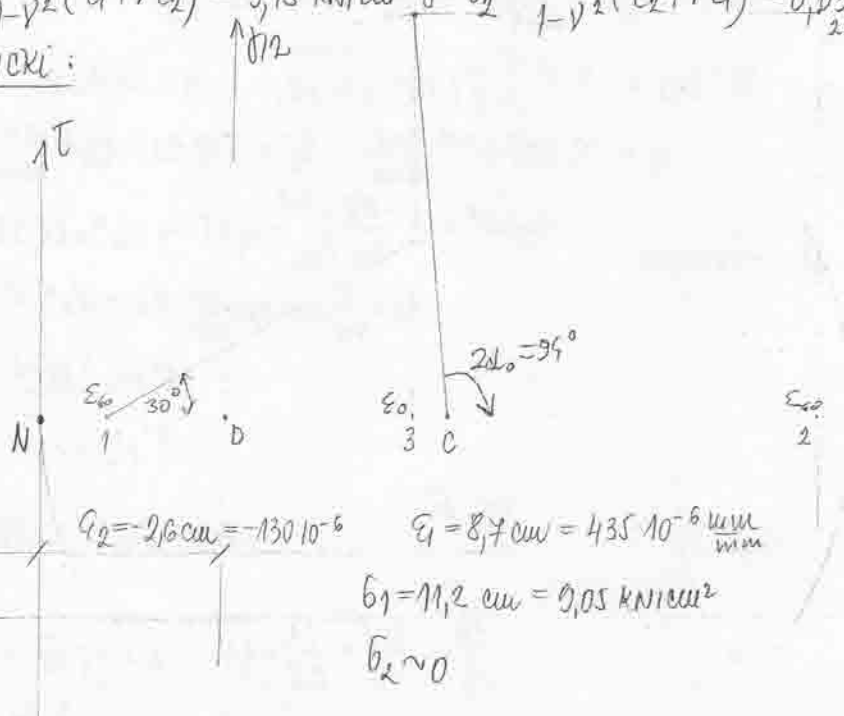
GRAFIČKI:

$$\epsilon_c = 50 \cdot 10^{-6} = 1 \text{ cm}$$

$$\epsilon_c = \frac{2,1 \cdot 10^{-1}}{1+0,3} \cdot 50 \cdot 10^{-6} = 0,808 \text{ kN/cm}^2 \approx 1 \text{ cm}$$

$$\frac{1+0}{1-0} = \frac{\bar{CN}}{\bar{CO}} = \frac{1,3}{0,7} = 1,857$$

$$\bar{CN} = 1,857 \cdot \frac{152}{50} = 5,64 \text{ cm}$$



$$\epsilon_2 = -26 \text{ cm} = -130 \cdot 10^{-6}$$

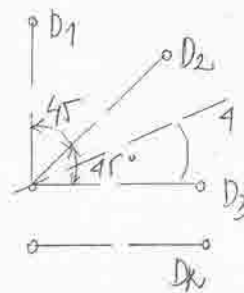
$$\epsilon_1 = 8,7 \text{ cm} = 435 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\sigma_1 = 11,2 \text{ cm} = 9,05 \text{ kN/cm}^2$$

$$\sigma_2 \approx 0$$

8) Na bet = l = m = 1710, u tacki A, put = m roz = t = mer = n = su dilataci = . Računski i grafički odrediti naponsko i deformacijsko stanje, a zatim grafički očitati vrednosti dilatacija i napona u pravcu 4-4. Merenje je izvršeno deformetrom Pfender (l = 100 mm).

Stanje	D1	D2	D3	D4
0	1695	1895	1620	2220
P	1655	1870	1610	2205
$\Delta = P - 0$	-40	-25	-10	-15
$\Delta - \Delta_k$	-25	-10	5	/
$\epsilon \cdot 10^6$	-250	-100	50	/



$$E_b = 0.3 \times 10^4 \text{ kN/cm}^2$$

$$\nu_b = 0.20 \text{ kN/cm}^2$$

$$\epsilon = (\Delta - \Delta_k) / l$$

$$l_{Pf} = 10 \cdot 10^{-6}$$

$\Delta_k$  kod Pfendera primeniti

$$\epsilon \cdot 10^6 = 10 \cdot (\Delta - \Delta_k)$$

Međutim znak dilatacijama da bi se

odgovorili minusa t = je sad + pritisak:

$$\epsilon_{D1} = \epsilon_{+45} = 250 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}, \quad \epsilon_{D2} = \epsilon_0 = 100 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}, \quad \epsilon_{D3} = \epsilon_{-45} = -50 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{1,2} = \frac{250 - 50}{2} \pm \frac{1}{2} \sqrt{(2 \cdot 100 - 250 + 50)^2 + (-50 - 250)^2} = 100 \pm \frac{1}{2} \sqrt{0^2 + (-300)^2} \rightarrow \epsilon_1 = 250 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}, \quad \epsilon_2 = -50 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\tan 2\alpha_0^* = \left| \frac{-300}{0} \right| = \infty \rightarrow \alpha_0^* = \frac{1}{2} \arctan \infty = 45^\circ, \quad \alpha_0 = 90 - 45 = 45^\circ$$

$$\sigma_1 = \frac{E}{1 - \nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{0.3 \cdot 10^4}{1 - 0.2^2} (250 - 0.2 \cdot 50) \cdot 10^{-6} = 0.75 \text{ kN/cm}^2$$

$$\sigma_2 = \frac{E}{1 - \nu^2} (\epsilon_2 + \nu \epsilon_1) = 0$$

$$\sigma_b = \frac{E}{1 + \nu} \cdot \frac{100 \cdot 10^{-6}}{r_E} \quad r_E = 1 \text{ cm} \approx 100 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\bar{C}N = \frac{1 + \nu}{1 - \nu} \cdot r_D = \frac{1.2}{0.8} \cdot 1 = 1.5 \text{ cm}$$

Očitavanje:

$$2\alpha_0 = 90^\circ \rightarrow \alpha_0 = 45^\circ$$

$$\epsilon_1 = 2.5 \text{ cm} \cdot 100 \cdot 10^{-6} = 250 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_2 = -0.5 \cdot 100 \cdot 10^{-6} = -50 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\sigma_1 = 3.025 = 0.75 \text{ kN/cm}^2$$

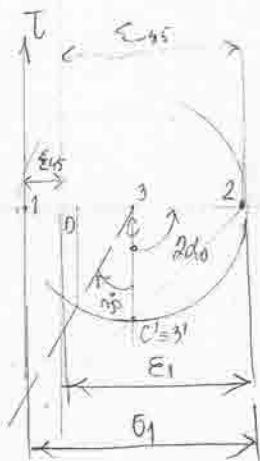
$$\sigma_2 = 0$$

$$\epsilon_{44} = 0.3 \cdot 100 \cdot 10^{-6} = 30 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

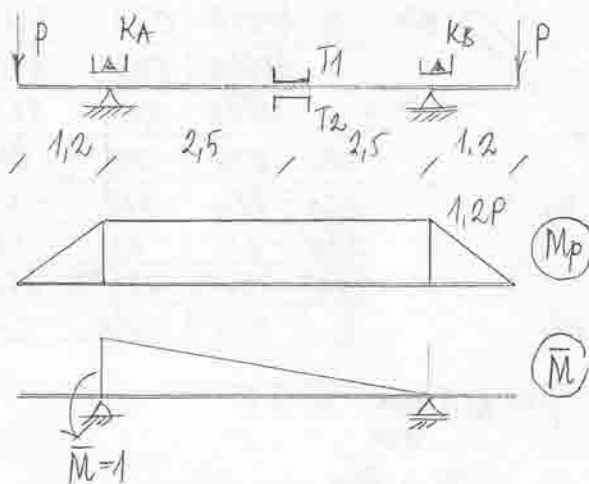
$$\nu/2 \cdot \epsilon_{44} = -1.25 \cdot 100 \cdot 10^{-6} = -125 \cdot 10^{-6}$$

$$\sigma_{44} = 0.8 \cdot 0.25 = 0.2$$

$$\epsilon_{41} = -1.25 \cdot 0.25 = -0.3125 \text{ kN/cm}^2$$



- 1) KOLIKI  $J = K_t$  (FAKTOR INSTRUMENTA) AKO  $K_t = 2,10$ , A MERNJA TRAKA  $T_2$  JE UKLONČENA KAO KOMPENZACIONA, PRI ISPITIVANJU NOSAČA PREMA SKICI, KLINOMETRI SU POKAZALI RAZIKU ČITANJA  $\Delta K = 284$ , A MERNJA TRAKA  $\Delta T = 160$ .



$$A\bar{C}K_A = A\bar{C}K_B$$

$$I_{300} \rightarrow A = 69 \text{ cm}^2$$

$$W = 653 \text{ cm}^3$$

$$J = 9800 \text{ cm}^4$$

$$EJ\varphi_A = \int M \bar{m} ds = \frac{l}{2} 1,2 P 1$$

$$p = \frac{2 EJ \varphi_A}{1,2 l}$$

+ KOMPENZACIONA TRAKA - OBICNA!

$$\varphi_A = \rho_K \cdot \Delta K = 1,06'' \cdot 284 = 301,04'' = 1,46 \cdot 10^{-3} \text{ rad}$$

$$P = \frac{2 \cdot 21 \cdot 10^4 \cdot 9800 \cdot 1,46 \cdot 10^{-3}}{120 \cdot 500} = 10,02 \text{ kN}$$

$$M_{T1} = 10,02 \cdot 1,2 = 12,024 \text{ kNm}$$

$$\bar{\sigma}_{T1} = \frac{12,024 \cdot 100}{653} = 1,841 \frac{\text{kN}}{\text{cm}^2}$$

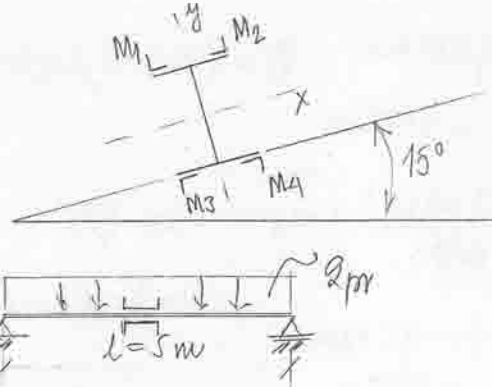
$$\epsilon_{T1} = \frac{\bar{\sigma}_{T1}}{E} = \frac{1,841}{21 \cdot 10^4} = 8,7683 \cdot 10^{-5} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{T1, \text{mer}} = \frac{K_t}{K_t} p_t \cdot A\bar{C} \quad \epsilon_{T1, \text{mer}} = \epsilon_{T1} \rightarrow 8,7683 \cdot 10^{-5} = \frac{K_t}{21} \cdot 0,5 \cdot 10^{-6} \cdot 160$$

$$\rightarrow K_t = 2,30$$

- 2) IZVRŠENO JE ISPITIVANJE POD PROBNIM OPTEREĆENJEM ROZNAČE I MERNJE SU DILATACIONE U PRESJECU U SREDINI RASPONA, PREMA SKICI. ODREDITI VELICINU PROBNOG OPT.

STANJE	0-1	0-2	0-3	"K"
0	10520	9710	8718	10535
2p	10468	9436	8992	10587
0	10520	9710	8718	10535
$\Delta \bar{C}_1$	-52	-274	274	52
$\Delta \bar{C}_2$	-52	-274	274	52
$\Delta \bar{C}_{sr}$	-52	-274	274	52
$\epsilon \cdot 10^6$	-52	-274	+274	52
$\bar{\sigma}$	-1,092	-5,754	5,754	1,092



$$I_{160}$$

$$J_{x-x} = 935 \text{ cm}^4$$

$$J_{y-y} = 54,7 \text{ cm}^4$$

$$W_{x-x} = 117 \text{ cm}^3$$

$$W_{y-y} = 14,8 \text{ cm}^3$$

$$\epsilon = \frac{K_t}{K_t} A\bar{C} \cdot p_t \quad p_t = 1 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\bar{\sigma} = \epsilon \cdot E \quad E = 21 \cdot 10^4 \text{ kN/cm}^2$$

$$\bar{\sigma}_{x-x} = \pm 3,423 \text{ kN/cm}^2$$

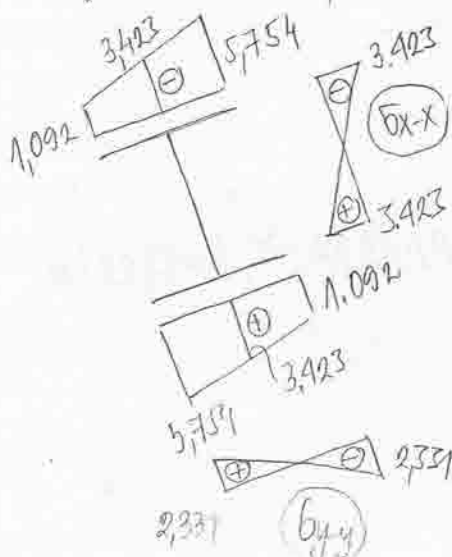
$$\bar{\sigma}_{y-y} = \pm 2,331 \text{ kN/cm}^2$$

$$\bar{\sigma}_{x-x} = \frac{M_{x-x}}{W_{x-x}} = \gamma M_x = 3,423 \cdot 117 = 400,491 \text{ kNm}$$

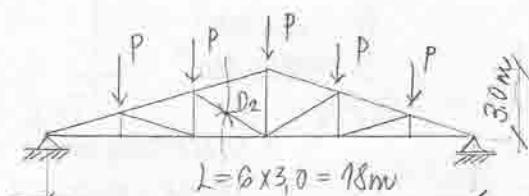
$$\bar{\sigma}_{y-y} = \frac{M_{y-y}}{W_{y-y}} \rightarrow M_y = 2,331 \cdot 14,8 = 34,499 \text{ kNm}$$

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{400,491^2 + 34,499^2} = 402 \text{ kNm} = 4,02 \text{ kNm}$$

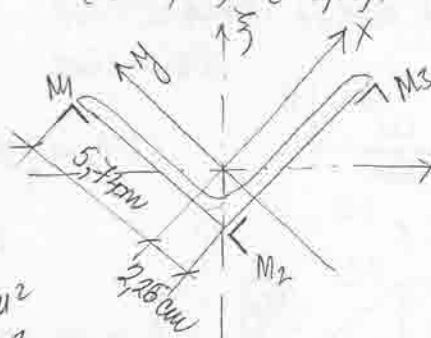
$$M = \frac{2 p l^2}{8} = \gamma 2 p l = \frac{4,02 \cdot 8}{5^2} = \frac{4,02 \cdot 8}{5^2} = 1,29 \sim 1,30 \frac{\text{kN}}{\text{m}}$$



- 3) NA STAPU D<sub>2</sub> ČELIČE KROVNÉ R=5 JK PRI PROBNOU OPTERČENJU MÉRENO JE D=FORMALCISKO STANJE MERNIM TRAKAMA. ODREDITI: A) SYE PR=SCN=81E U STAJM; B) ODREDITI VELICINU PROBNOU OPT P (K<sub>i</sub>=2,00, K<sub>t</sub>=2,15).



L 80 x 80 x 8  $E_c = 2,1 \cdot 10^4 \text{ kN/cm}^2$   
 $J_x = J_y = 72,3 \text{ cm}^4$   $J_z = 115,0 \text{ cm}^4$   
 $J_\eta = 29,6 \text{ cm}^4$   $A = 12,3 \text{ cm}^2$

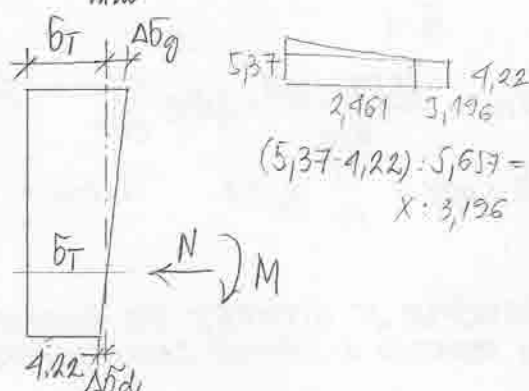
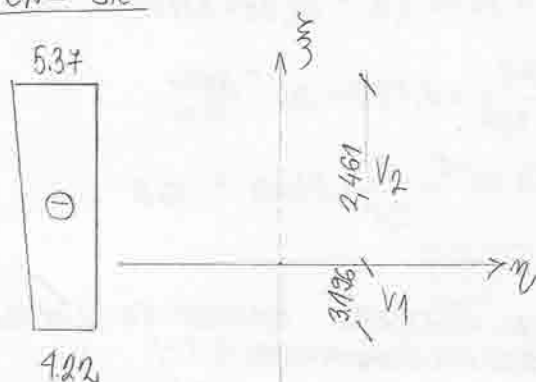


STANE	M1	M2	M3
0	11920	9130	8150
P	11646	8914	8476
$\eta$ 0	11922	9130	8152
$\Delta C_1$	-274	-216	-274
$\Delta C_2$	-276	-216	-276
$\Delta C_{\eta}$	-275	-216	-275
$E \times 10^9$	-255,81	-200,93	-255,81
$\bar{b}$	-5,37	-4,22	-5,37

$$\epsilon = \frac{k_i}{k_t} p \cdot \Delta \bar{c}$$

$$\frac{k_i}{k_t} = \frac{2,00}{2,15} \quad p = 1 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \quad \bar{b} = \epsilon \cdot E$$

- PR=SCN=81E



$$(5,37 - 4,22) \cdot 5,657 = 6,196$$

$$V_1 + V_2 = 8 \cdot \cos 45 = 5,657 \text{ cm} \quad V_2 = 5,657 - 3,196 = 2,461 \text{ cm}$$

$$V_1 = 2,26 \cdot \frac{1}{\cos 45} = 3,196 \text{ cm}$$

$$\bar{b}_T = -4,22 + \frac{(-5,37 + 4,22)}{5,657} \cdot 3,196 = -4,22 - 0,65 = -4,87 \text{ kN/cm}^2$$

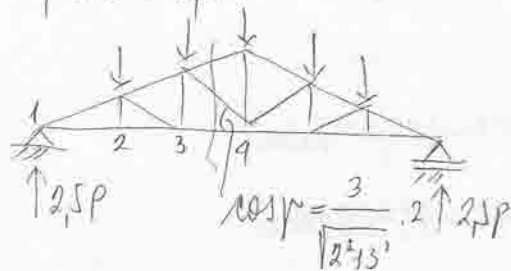
$$\Delta \bar{b}_d = +0,65 \text{ kN/cm}^2$$

$$\Delta \bar{b}_g = -5,37 - (-4,87) = -0,5 \text{ kN/cm}^2$$

$$M_\eta = \Delta \bar{b}_d \cdot \frac{J_\eta}{V_1} = 0,65 \cdot \frac{29,6}{3,196} = 6,02 \text{ kNcm} \quad M_z = 0$$

$$N = \bar{b}_T \cdot A = -4,87 \cdot 12,3 \text{ cm}^2 = -59,901 \text{ kN}$$

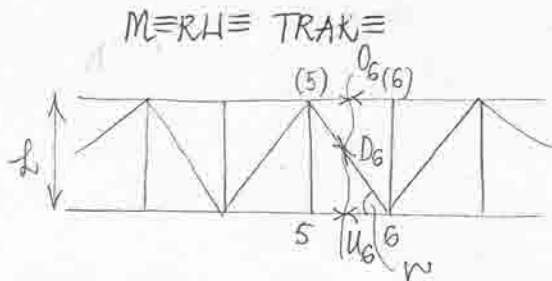
- PROBNO OPT.



$$\frac{M_1}{L_1} = \frac{1}{3} (2,5P \cdot 9 - 3P \cdot 3) = 4,5P$$

$$\frac{M_3}{L_3} = \frac{1}{2} (2,5P \cdot 6 - 3P) = 6P$$

$$D_2 = -59,901 = (4,5P - 6P) \cdot 1,202 = \gamma \quad P = 37,22 \text{ kN}$$



$$\sec \phi = \frac{1}{\cos \phi}$$

$$Q_6 = -\frac{M_6}{L_6}$$

$$U_6 = \frac{M(5)}{L_5}$$

$$D_6 = \left( \frac{M_6}{L_6} - \frac{M(5)}{L_5} \right) \sec \phi_{D_6}$$

RAČUNSKO  
VR=DNOSTI

RAZUK U RAČUNSKIM VR=DNOSTIMA I

M=RNIM;

$$\Delta U_6 = \frac{U_{6, \text{rač}} - U_{6, \text{mer}}}{U_{6, \text{rač}}} \cdot 100$$

PRKO SU RAZUK  
DO 10%

M=RNJ JE  
DOBRO!

PF=ND=R  $p_{pf} = 10 \cdot 10^{-6} \text{ mm/mm}$   $\oplus ZAT=ZANJ$   
 $\epsilon = \Delta \bar{c} \cdot p_{pf}$

M=RN= TRAK=

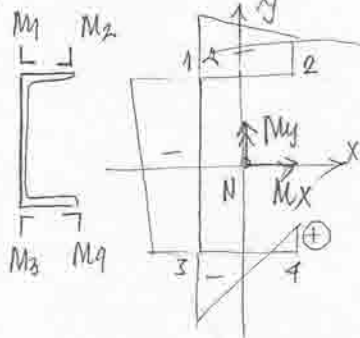
$$p = 1 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \leftarrow \text{OBICNA}$$

$$p = 95 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} \leftarrow \text{KOMPENZACIONA}$$

$$\epsilon = \Delta \bar{c} \cdot \frac{k_i}{k_t} \cdot p$$

$$\sigma = \epsilon \cdot E$$

B=T. PR=SK  $\rightarrow G=OM$ . KAKAK.  $(A_6) y_T, J_6, W_{6g}, W_{6d}$   
- ODR=O. PR=SE CNH 81A  $T \rightarrow \sigma_N, M$



$\sigma_{M1}, \sigma_{M2}, \sigma_{M3} \rightarrow N, M_x, M_y$   
ZAT. +  
 $\sigma_{M4} \rightarrow \text{PROVERA}$

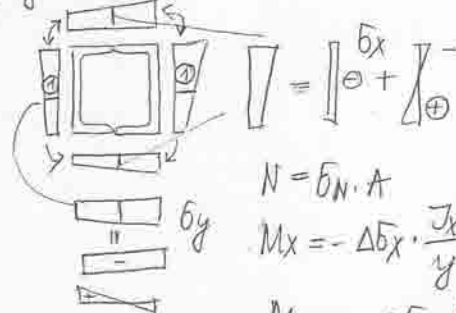
$$N = \sigma_N \cdot A$$

$$M_g = \sigma_g \cdot W_g$$

$$M_d = \sigma_d \cdot W_d$$

$M_g \approx M_d \rightarrow$

$$M = \frac{1}{2} (M_g + M_d)$$



$$N = \sigma_N \cdot A$$

$$M_x = -\Delta \sigma_x \cdot \frac{J_x}{y}$$

$$M_y = +\Delta \sigma_y \cdot \frac{J_y}{x}$$

D=FORMETAR  $\rightarrow$  LABISKON  $\ell = 250 \text{ mm}$

$$p = 4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\oplus \text{ PRITSAK} \rightarrow \text{UV.} \quad \epsilon = -p \Delta \bar{c}$$

$$\sigma = \epsilon \cdot E$$

$$D1 D2 D3 D4 DK \quad \epsilon = (\Delta \bar{c} - A_k) p_L$$

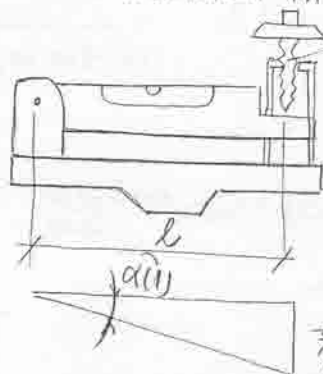


# INSTRUMENTI

$$I_x = \frac{4}{36} a l^3 \quad I_y = \frac{4}{36} a^3 l$$

1) NOVEMBAR 2004.

KONSTRUKCIJA KLINOMETRA SLEDEĆIH KARAKTERISTIKA: A) 400 MIKROMETARSKOG ZAVRTANJA JE HILADITI D=0 RAZMATKA L IZMEĐU ŽIGUBA I OSE MIKROMETARSKOG ZAVRTANJA. B) ODREDI TI IMZINU (l) I KO PUDATKE KLINOMETRA IZNOSI 1" PRI OSTALIM PODREBNIM KARAKTERISTIKAMA KAO POD a).



MIKROMETARSKA ZAVRTANJA a) STANDARDNO  $l = 195 \text{ mm}$   $n = 250$  PODELA

$$k = \frac{l}{1000}$$

$$\alpha(1) = \frac{k}{n \cdot l} = \frac{l}{1000 \cdot l \cdot n} = \frac{1}{1000 \cdot 250} = 4 \cdot 10^{-6}$$

$$\alpha''(1) = 4 \cdot 10^{-6} \frac{180 \cdot 3600}{\pi} = 0,825''$$

b)  $\alpha''(1) = 1''$  STANDARDNO:  $n = 250$   $k = 0,25 \text{ mm}$

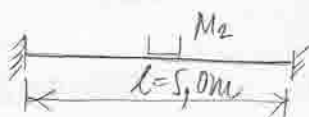
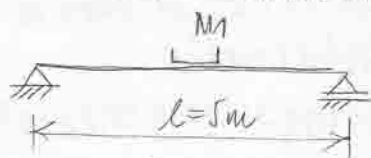
$$\alpha(1) = \frac{1''}{3600 \cdot 180} = \frac{k}{n \cdot l} = \frac{0,25}{250 \cdot l}$$

$$4,848 \cdot 10^{-6} = \frac{0,25}{250 \cdot l}$$

$$l = \frac{0,25}{250 \cdot 4,848 \cdot 10^{-6}} = 206,3 \text{ mm}$$

2) APRIL 2004.

KOLIK JE RAZLIKE ČITANJA DOKUŠENJU NA MERENOM MOSTU AKO JE AKTIVNA MERNA TRAKA ZALPLENA NA NOSIOTIMA PREMA SKICI, U OBA SLUCAJA NOSIOTI SU PRESEKA  $I 160$  ( $A = 22,8 \text{ cm}^2$ ), I IZLOŽENI SU RAZNOMERNOM ZAGREVANJU OD  $t = 15^\circ \text{C}$ . DATI OBJASNJENJE



$$K_i = K_t$$

$$\alpha_t = 1 \cdot 10^{-5} / ^\circ \text{C} \quad t = 15^\circ \text{C}$$

$$\epsilon = \alpha_t \cdot t = 15 \cdot 10^{-5} = 150 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}} = \frac{K_i}{K_t} \cdot \Delta \bar{c} \cdot \rho \cdot 1 \cdot 10^{-6} \quad [\Delta \bar{c} = 150]$$

Slučaj a)  $\Rightarrow \Delta \bar{c} = 0 \rightarrow$  jer će kompenzacija automatski oduzeti uticaj temperature

Slučaj b)  $\Rightarrow \Delta \bar{c} = -150$  jer na aktivnoj traci  $M_2$  je sprečena dilatacija, ali kompenzacija odustaje uticaj temper.  $t^\circ$  i dobije se očitaje

$$\Delta \bar{c} = \bar{c}_{0t} - \bar{c}_t$$

$$a) \Delta \bar{c} = \bar{c}_t - \bar{c}_t = 0$$

$$b) \Delta \bar{c} = \bar{c} - \bar{c}_t = 0 - 150 = -150$$



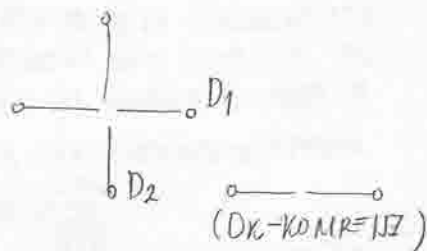
3) IZVRŠNO JE MERENJE NA BETON DILATAIONA U ZORNOGHALNA PRAVCA  $D_1$  I  $D_2$ , KOJI SU UEDNO I TRAJKODRIJE GL. NAPONA, INSTRUMENTOM LABISTON ( $L=250\text{ mm}$ ). ODRDITI NAPONSKO STANJE U ISPITIVANOJ TACKI.

STANJE	$D_1$	$D_2$	$D_k$
Jun 2003	1950	1740	1910
Maj 2003	2060	1700	1885
$\Delta$	110	-40	-25
$\Delta - A_k$	135	-15	
$\varepsilon$	$540 \cdot 10^{-6}$	$-60 \cdot 10^{-6}$	

$$E_b = 0,3 \cdot 10^4 \text{ KN/cm}^2$$

$$\nu_b = 0,20$$

$$\rho_L = 4 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

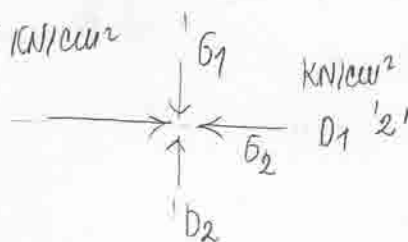


PRINCIPI  $D_1$  I  $D_2$  SU GLAVNI PRAVCI PA JE:  $\varepsilon_{D1} = 540 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$ ;  $\varepsilon_{D2} = -60 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$

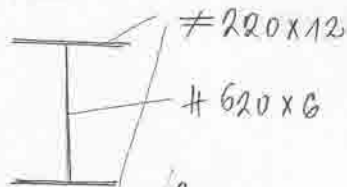
$$\sigma_1 = \frac{E}{1-\nu^2} (\varepsilon_1 + \nu \varepsilon_2) = \frac{0,3 \cdot 10^4}{1-0,2^2} (540 - 0,2 \cdot 60) \cdot 10^{-6} = 0,15 \text{ KN/cm}^2$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\varepsilon_2 + \nu \varepsilon_1) = \frac{0,3 \cdot 10^4}{1-0,2^2} (-60 + 540 \cdot 0,2) \cdot 10^{-6} = 1,65 \text{ KN/cm}^2$$

+ PRITISAK



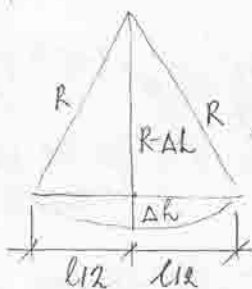
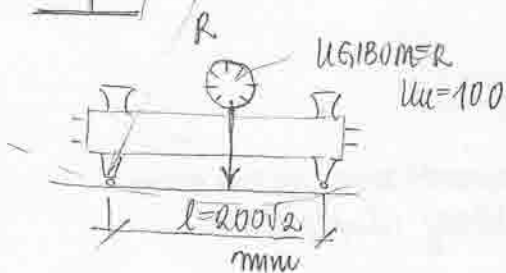
4) NA CILICHOJ GRADI, MERENJA JE PROMENA KRIVINE U MERODATVOM '1' PRESSEKU. REZULTATI MERENJA -> TABLICA, MERENJE IZVRŠNO STANDARTNIM MEHANICKIM KRIVINOMEROM ( $U_u = 100$ ). ODRDITI PRESSEKNU ILLI KOJA ODGOVARA OVOJ DEFORMACIJI, KOLIKO BI CITAJE BILLO NA UGIBOMERU DA JE UPOTREBIEN UGIBOMER KU=6464 U<sub>u</sub>=1000.



STANJE	$U_1$
0	0236
P	0238

$$G.K.P. \quad J = \frac{1}{12} 0,6 \cdot 62^3 + 2 \frac{1}{12} 1,2^3 \cdot 22 + 2 \cdot 22 \cdot 1,2 \cdot 31,6^2 = 64646,7 \text{ cm}^4$$

$$EJ = 2,1 \cdot 10^4 \cdot 64646,7 = 135758,08 \cdot 10^9 \text{ KN/cm}^2$$



$$R^2 = (R - \Delta h)^2 + (l/2)^2$$

$$R^2 = R^2 - 2R\Delta h + \Delta h^2 + \frac{l^2}{4}$$

$$2R\Delta h = \Delta h^2 + 20000$$

$$R\Delta h = 10^4 \rightarrow R = \frac{10^4}{\Delta h}$$

poluprečnik krivine

DEFORMACIJA  $\Delta \bar{c}$

$$\Delta h = (238 - 236) \frac{1}{100} = 0,02 \text{ mm} = 2 \cdot 10^{-3} \text{ cm}$$

$$R = \frac{10^4}{0,02} = 50 \cdot 10^4 = 500 \text{ m}$$

$$\chi = \frac{1}{R} = 0,002 \frac{1}{\text{m}}$$

PRISSEKNA SNA:

$$\chi = -\frac{M}{EJ} \rightarrow M = -\chi EJ = -0,002 \cdot 135758,08$$

$$M = -271,52 \text{ KNm}$$

CITANJE:

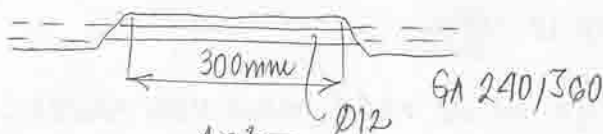
$$\Delta h = (\bar{c}_p - \bar{c}_0) \frac{1}{u_u} = \gamma$$

$$\bar{c}_p = \Delta h \cdot u_u + \bar{c}_0$$

$$\bar{c}_p = 0,02 \cdot 1000 + 236 = 256$$

$$\bar{c}_p = 0256$$

- 6) U CILINDRIČNOM ODRŽIVANJA NAPONSNOG STANJA U PRISILNOJ ABLEMTA, KOJI JE VEĆ BIO IZLOŽEN NAPREZANJU, IZVRŠENO JE OGOLOTVANJE ARMATURE I MERENJE FREKVENCIJE SLOBODNOG OSCILOVANJA POJEDINIH SIPKE Ø12 NA DUŽINI SLOBODNOG OSCILOVANJA OD 300 mm. IZMERENA JE FRKV. OD  $f_0 = 300 \text{ Hz}$ . KOLIKI JE NAPON U ARMATURI?



$$E_c = 21 \cdot 10^4 \text{ kN/cm}^2$$

$$I_c = 78,50 \text{ cm}^4$$

$$f_0 = 300 \text{ Hz}$$

$$\phi 12 \text{ mm} \rightarrow A = \frac{\pi \cdot 12^2}{4} = 113 \text{ mm}^2$$

$$J = \frac{\pi \cdot 12^4}{64} = 0,101736 \text{ cm}^4$$

$$E_c = 21 \cdot 10^4 \text{ kN/cm}^2$$

$$I_c = 78,50 \cdot 10^{-6} \text{ m}^4$$

FRKVENCIA SLOBODNOG OSCILOVANJA SIPKE (ZATEGNUTE) KOJA IMA KRUTOŠT NA SAVIJANJE:

$$f_0^2 = \frac{b \cdot g}{4l^2 I_c} + \frac{\pi^2 E_c J_g}{4l^4 A I_c} = \gamma$$

$$b = 4 \frac{I_c}{g} \left[ f_0^2 - \frac{\pi^2 E_c J_g}{4l^4 A I_c} \right] \cdot l^2 = 0,1 \cdot l^2 \left[ f_0^2 - \frac{\pi^2 E_c J_g}{4l^4 A I_c} \right]$$

$$b = 32 \cdot 10^{-7} \cdot 30^2 \left[ 300^2 - \frac{\pi^2 \cdot 21 \cdot 10^4 \cdot 0,101736 \cdot 981}{4 \cdot 30^4 \cdot 113 \cdot 78,50 \cdot 10^{-6}} \right] = 32 \cdot 10^{-7} \cdot 30^2 [300^2 - 71973]$$

$$b = 5,19 \text{ kN/cm}^2$$

- 7) MEHANIZAM ZA UVEĆANJE ČITANJA KOD DATOG UGIROMERA JE SASTAVLJEN OD 4 PARA ZUPČANIKA. ODRŽITI: a) ODRŽITI POMIČNIK ZUPČANIKA TRČEĆI PARA ( $r_6/r_5 = ?$ ) AKO SU ODNOSI  $r_2/r_1 = 2$ ;  $r_4/r_3 = 5$  I  $r_8/r_7 = 10$  I AKO JE BROJ PODELA NA BROJČANIKU VEĆE KAZALJKE 100, A BROJ PODELA NA BROJČANIKU MALE KAZALJKE 20, PRI ČEMU JE OPSJE INSTRUMENTA 2 mm

b) OPSJE INSTRUMENTA AKO JE  $r_2/r_1 = r_6/r_5 = 2$ ;  $r_4/r_3 = r_2/r_1 = 5$ , A NADVEĆA MOGUĆA RAZLIKA ČITANJA KOJA SE MOŽE OSTVARITI ISTA JE KAO ZA SLUČAJ KONSTRUKCIJE UGIROMERA PREMA VREDNOSTIMA U TAČKI A).

$$u_u = \frac{r_2}{r_1} \frac{r_4}{r_3} \frac{r_6}{r_5} \frac{r_8}{r_7}$$

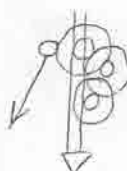
$$a) \Delta c_{\max} = 2000$$

$$u_{\max} = 0_u = 2 \text{ mm} \quad \left. \begin{array}{l} \Delta c_{\max} = 2000 \\ u_{\max} = 0_u = 2 \text{ mm} \end{array} \right\} \rightarrow u_u = \frac{\Delta c_{\max}}{0_u} = 1000$$

$$1000 = 2 \cdot 5 \left( \frac{r_6}{r_5} \right) \cdot 10 \rightarrow \frac{r_6}{r_5} = \frac{1000}{100} = 10$$

$$b) u_u = 2 \cdot 5 \cdot 2 \cdot 5 = 100$$

$$\Delta c_{\max} = 2000 \quad \left. \begin{array}{l} u_u = 100 \\ \Delta c_{\max} = 2000 \end{array} \right\} \rightarrow 0_u = \frac{\Delta c_{\max}}{u_u} = \frac{2000}{100} = 20 \text{ mm}$$



5) JANUAR 2004.

TENZOMETAR "GAUL=0" BAZ= 20 mm, MOŽE ISTOVREMENO DA IMA PODATAK  $\epsilon(1) = 50 \times 10^{-6} \frac{\text{mm}}{\text{mm}}$  I  $\epsilon(1) = 250 \times 10^{-6} \frac{\text{mm}}{\text{mm}}$ . KAKO SE POSTIŽE KOD OVOG TIPIA INSTRUMENTA OVA PROMENA VELICINE PODATKA I ZA STA BI SE MOGLA ISKORISTITI.

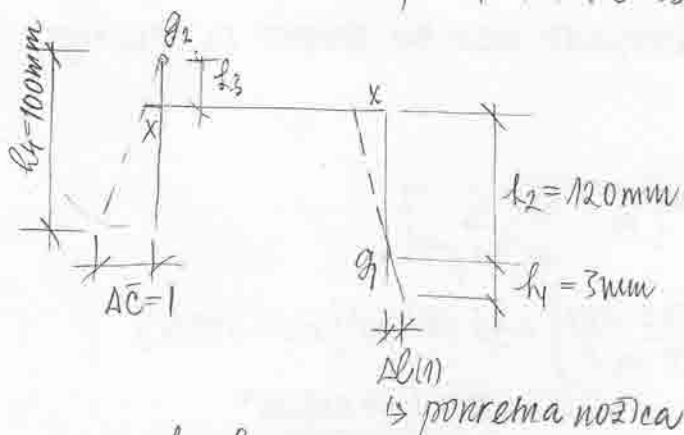
- STANDARDNO UVETAŃJE JE  $U_T = 1000$

$$l = 20 \text{ mm} \rightarrow \epsilon(1) = \frac{\Delta l(1)}{l} = \frac{1/1000}{20} = 50 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

- DRUGA MOGUĆNOST JE SMANJENJE UVETAŃJA NA  $U_T' = 200$ , PROMENOM POLOŽAJA POSEBNOG DUGMETA NA TENZOMETRA:

$$l = 20 \text{ mm} \rightarrow \epsilon(1) = \frac{\Delta l(1)}{l} = \frac{1/200}{20} = 250 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

ŽNAČI DA PROMENA PODATKA POTIČE OD PROMENE UVETAŃJA:



$$\frac{\Delta l(1)}{l_1} = \frac{x}{l_2} \Rightarrow x = \frac{\Delta l(1)}{\frac{l_1}{l_2}}$$

$$\frac{x}{l_3} = \frac{\Delta \bar{c}}{l_4}$$

$$\Delta \bar{c}(1) = \frac{x}{\frac{l_3}{l_4}} = \frac{\Delta l(1)}{\frac{l_1}{l_2} \cdot \frac{l_3}{l_4}}$$

$$\Delta l(1) = \frac{\Delta \bar{c}(1)}{\frac{l_2}{l_1} \cdot \frac{l_4}{l_3}} = \frac{\Delta \bar{c}}{U_T}$$

$$U_T = \frac{l_2}{l_1} \cdot \frac{l_4}{l_3} = \frac{120}{100} \cdot \frac{100}{3} = 1000$$

$$U_T' = \frac{l_2'}{l_1'} \cdot \frac{l_4}{l_3} = \frac{l_2'}{l_1'} \cdot 25 = 200 \Rightarrow \frac{l_2'}{l_1'} = 8$$

MERENJE SE POLOŽAJ ZGLUBA  $g_1$  (PROMENOM NA DUGMETSU) TAKO DA ODNOS KRAKOVA PODRETNJE POLUGE 8.

- OVU OSOBINU MOŽE MO ISKORISTITI KOD MERENJA VELIKIH DILATACIJA, JER SE POVEĆAVA OPSEG MERENJA:  $\Delta \bar{c}_{max} = 40$

- STANDARDAN OPSEG MERENJA:  $0 = 40 \cdot 50 \cdot 10^{-6} = 2000 \cdot 10^{-6}$

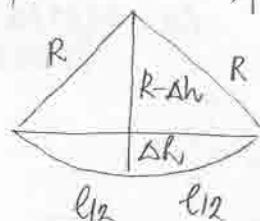
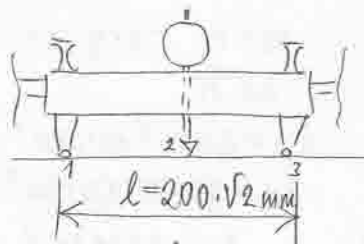
- NOVI

- II

$$0' = 40 \cdot 250 \cdot 10^{-6} = 10000 \cdot 10^{-6}$$

- 8) NA AB PLOČI SU KRIVINOMEROM MERENI PROMENI KRIVINE U DVA DRUGI PRAVCA, OBJASNITI KONSTRUKCIJU INSTRUMENTA I ODREDITI MERENI MOMENTI SAVIJANJA PLOČE, AKO SU RAZLIKE OTANJA NA UGIROMERU ( $f_m = 0,01 \text{ mm}$ ) KRIVINOMERA  $\Delta \bar{c}_x = 3, \Delta \bar{c}_y = 6$ .  
 $(E_b = 0,3 \cdot 10^9 \text{ kN/m}^2, \nu = 0,18, d_{pl} = 12 \text{ cm})$ .

a) KONSTRUKCIJA KRIVINOMERA; PODATAK



$$f_m = 0,01 \text{ mm}$$

PR. - TACK = 1, 2, 3 IMAJU KONSTANTNU

KRIVINU (DEFORMACIONA LINIJA → KRUG)

$$R^2 = \left(\frac{l}{2}\right)^2 + (R - \Delta h)^2$$

$$R^2 = 2 \cdot 10^9 + R^2 - 2RA\Delta h + \Delta h^2$$

$$R = \frac{10^9}{\Delta h} \leftarrow \text{POLUPRČNIK KRIVINE}$$

$$\Delta \bar{c}_x = 3, \Delta \bar{c}_y = 6$$

$$\Delta h_x = 3 \cdot 0,01 = 0,03 \text{ mm} \quad \Delta h_y = 6 \cdot 0,01 = 0,06 \text{ mm}$$

$$\kappa_x = \frac{\partial^2 W}{\partial x^2} = \frac{1}{R_x} = \frac{\Delta h_x}{10^9} = \frac{3 \cdot 10^{-2}}{10^9} = 3 \cdot 10^{-6} \frac{1}{\text{mm}} = 3 \cdot 10^{-3} \frac{1}{\text{m}}$$

$$\kappa_y = \frac{\partial^2 W}{\partial y^2} = \frac{1}{R_y} = \frac{\Delta h_y}{10^9} = \frac{6 \cdot 10^{-2}}{10^9} = 6 \cdot 10^{-6} \frac{1}{\text{mm}} = 6 \cdot 10^{-3} \frac{1}{\text{m}}$$

$$K_s = \frac{E d_{pl}^3}{12(1-\nu^2)} = \frac{0,3 \cdot 10^8 \cdot 0,12^3}{12(1-0,18^2)} = 4345,34 \text{ kNm}$$

$$M_x = -K_s \left( \frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} \right) = -4345,34 (3 + 0,18 \cdot 6) \cdot 10^{-3} = -17,73 \text{ kNm}$$

$$M_y = -K_s \left( \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} \right) = -4345,34 (6 + 0,18 \cdot 3) \cdot 10^{-3} = -28,92 \text{ kNm}$$

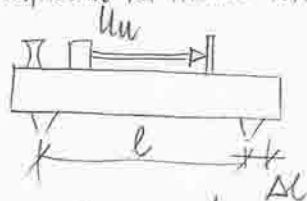
- 9) ODABRATI DEFORMETAR NAJVEĆE TAČNOSTI AKO SU NA RASPOLAGANJU 3 DEFORMETRA SLIDEĆIH KARAKTERISTIKA:

1.  $l = 100 \text{ mm}$  (DUŽINA BAZE)  $u_u = 1000$  (UVEĆANJE UGIROMERA)

2.  $l = 250 \text{ mm}$   $u_u = 500$

3.  $l = 500 \text{ mm}$   $u_u = 100$

UGIROMERI IMAJU ISTI BROJ PODELA NA OROJČANIKU.



$$\epsilon = \frac{\Delta l}{l} \quad P_d = E(\eta) \quad \Delta l(\eta) = \frac{\Delta \bar{c}}{u_u} = \frac{1}{u_u}$$

$$\epsilon(\eta) = \frac{\Delta l(\eta)}{l} = \frac{1}{u_u l}$$

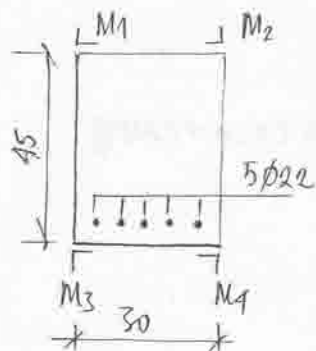
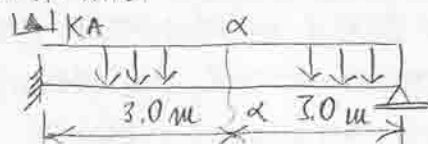
D.1.  $\epsilon(1) = \frac{1}{100 \cdot 1000} = 10 \cdot 10^{-6}$

D.2.  $\epsilon(2) = \frac{1}{500 \cdot 250} = 8 \cdot 10^{-6}$

D.3.  $\epsilon(3) = \frac{1}{100 \cdot 500} = 20 \cdot 10^{-6}$

10) D=2000  
 NA AB = LEMENJU, MERENJE SU MERHIM T. LOKALNE DEFORM U PRESKU  $\alpha-\alpha$   
 I PROMENA NAGIBA DEFORM. LINIJE U OBLONCU A. NAČI:

- A) NAPONSKO STANJE U PRESKU  $\alpha-\alpha$   
 B) OPTEREĆENJE  $Q$   
 C) MOMENT. EL. UKLOSTENJA U CVRU A



ST	M1	M2	M3	M4	KA
0	10153	12654	11136	11285	0.056
2	9780	12277	11766	11925	0.789
2-0	-373	-377	630	640	128
E	-373	-377	630	640	135.68"
$E_{8r}$	-375		635		0.00066
$b_{8r}$	-1.125		13.335		

$K_i = K_e \cdot C_{240/360}$   
 $M_{B30}$

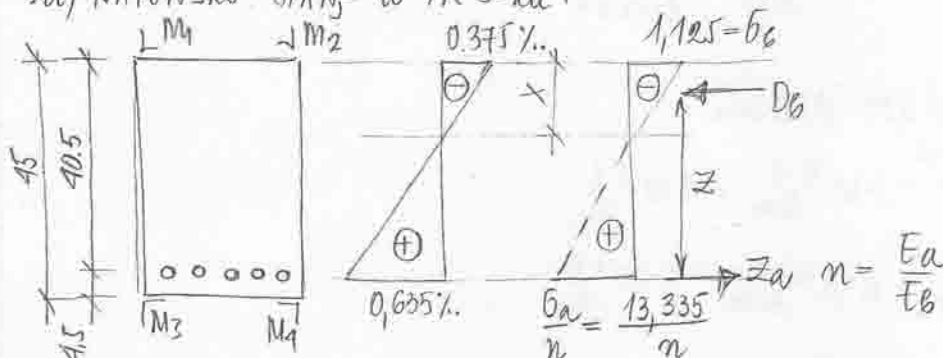
$E_c = 2.1 \cdot 10^9 \text{ KN/cm}^2$

$E_b = 0.3 \cdot 10^9 \text{ KN/cm}^2$

$A_a = 19.01 \text{ cm}^2$   
 $(5 \phi 22)$



a) NAPONSKO STANJE U PRESKU:



$x = \frac{E_b}{E_a + E_b} \cdot l = \frac{0.375}{0.635 + 0.375} \cdot 40.5$   
 $x = 15.04 \text{ cm}$

$D_b = \frac{1}{2} b_b \cdot x = -\frac{1}{2} \cdot 1.125 \cdot 15.04 \cdot 30 = -253.8 \text{ kN}$   
 $Z_a = b_a A_a = 13.335 \cdot 19.01 = 253.5 \text{ kN}$   
 $\sum N = 0$   
 CISTO SAVIJANJE

b) KRATK UNUTRASNJIH SILA:  $z = l - \frac{x}{3} = 40.5 - \frac{1}{3} \cdot 15.04 = 35.05 \text{ cm}$

MOMENT U PRESKU:  $M = Z_a \cdot z = 253.5 \cdot 35.05 \cdot 10^{-2} = 89.59 = 90 \text{ kNm}$

DVA GRAMU MOMENTATA:  $\frac{Ql^2}{8} = \frac{MA}{2} + M$

$MA = \frac{Ql^2}{8} - \frac{3EJ}{l} \tan \alpha = \frac{Ql^2}{8} - \frac{3.0 \cdot 3 \cdot 10^8}{6.0} \cdot \frac{1}{12} \cdot 0.3 \cdot 0.45^3 \cdot 0.00066 = \frac{Ql^2}{8} - 22.5$

$\frac{Ql^2}{8} = \frac{Ql^2}{16} - \frac{22.5}{2} + 90 \rightarrow \frac{Ql^2}{16} = 78.75 \rightarrow Q = \frac{78.75 \cdot 16}{36} = 35 \text{ kN/m}$

c)  $MA = \frac{1}{8} \cdot 35 \cdot 36 - 22.5 = 135 \text{ kNm}$

\* NA = LICNOM STUBU I SPIN, CHU M B = FONOM M = R = N = SU M = R NIM TRAKAMA  
LOKAIN = DEFORMATIJE PR = NA SKIC ODREDITI PR = S = CN = SILE U STUBU PR = NA  
R = ZULFATNA M = R = NA DATUM U TABLICI.

STANJE	M1	M2	M3	M4
O	11375	10220	9967	10122
P	11350	10189	9938	10099
O	11372	10216	9970	10125
P-OI	-25	-31	-29	-23
P-OII	-22	-27	-32	-26
ΔSR	-23,5	-29	-30,5	-24,5
E 10 <sup>6</sup>	-25,974	-32,053	-33,711	-27,079

$$E_b = 0,35 \times 10^4 \text{ KN/cm}^2$$

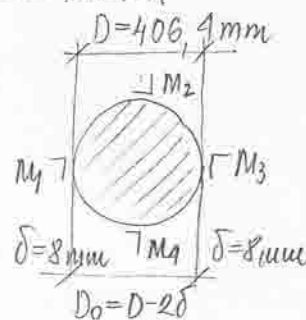
$$E_c = 2,10 \times 10^4 \text{ KN/cm}^2$$

$$Y_b = 0,18 \text{ cm}^2 \quad Y_c = 0,30$$

$$K_i = 2,10 \quad K_t = 1,90$$

$$J_0 = D^4 \pi / 64$$

$$E = \frac{K_i}{K_t} \cdot p \cdot A_c$$



G = OM. KARAKT = RISTIK = :

$$A_{bi} = A_b + \frac{E_c}{E_b} A_c \quad J_{bi} = J_b + \frac{E_c}{E_b} J_c$$

$$D = 40,64 \text{ cm} \quad D_0 = D - 2\delta = 40,64 - 2 \cdot 0,8 = 39,04 \text{ cm}$$

$$A_b = \frac{D_0^2 \pi}{4} = \frac{39,04^2 \pi}{4} = 1197,04 \text{ cm}^2 \quad A_c = (D^2 - D_0^2) \frac{\pi}{4} = 100,129 \text{ cm}^2$$

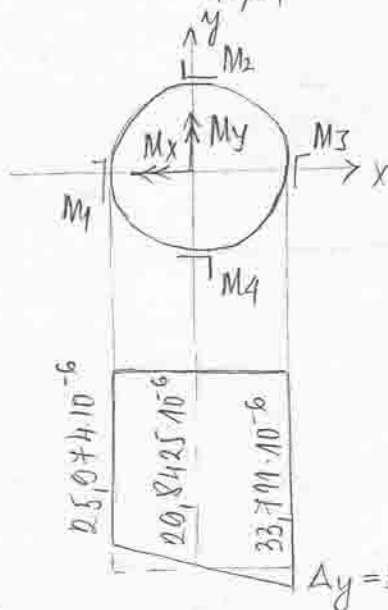
$$J_b = \frac{D_0^4 \pi}{64} = \frac{39,04^4 \pi}{64} = 114027,377 \text{ cm}^4$$

$$J_c = (D^4 - D_0^4) \frac{\pi}{64} = 19873,803 \text{ cm}^4$$

$$A_i = 1197,04 + \frac{2,1}{0,35} \cdot 100,129 = 1707,814 \text{ cm}^2$$

$$J_i = 114027,377 + \frac{2,1}{0,35} \cdot 19873,803 = 233270,735 \text{ cm}^4$$

$$W_i = \frac{2J_i}{D} = \frac{2 \cdot 233270,735}{40,64} = 11479,850 \text{ cm}^3$$



$$\begin{aligned} & 32,053 \cdot 10^{-6} \\ & 20,566 \cdot 10^{-6} \\ & 27,079 \cdot 10^{-6} \\ & \Delta X = 2,487 \cdot 10^{-6} \end{aligned}$$

$$E_{SR} = \frac{20,566 + 20,8425}{2} \cdot 10^{-6}$$

$$E_{SR} = 20,704 \cdot 10^{-6}$$

$$E_{\Delta X} = 2,487 \cdot 10^{-6} \quad E_{\Delta Y} = 3,8685 \cdot 10^{-6}$$

$$\Delta Y = 3,8685 \cdot 10^{-6}$$

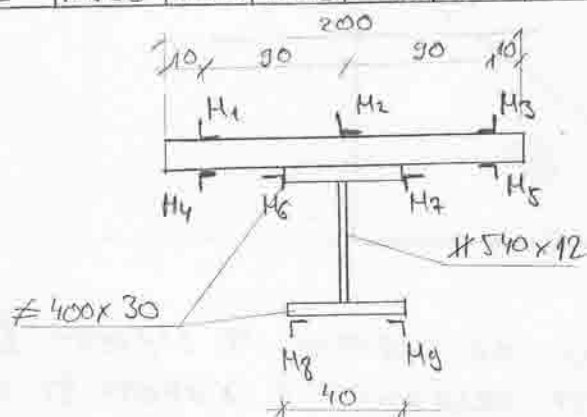
PR = S = CN = SILE :

NSMBA =



1<sup>o</sup> ПЕРНИИ ТРАКАНА ЈЕ ИСПИТИВАНО ДЕФОРМАЦИЈСКО СТАЊЕ СПРЕГНУТОГ НОСАЧА МЕЂУСПРАТНЕ КОНСТРУКЦИЈЕ ПОД ПРОБНИМ ОПТ. ПРЕМА СКИЦИ. НА ОСНОВУ РЕЗУЛТАТА МЕРЕЊА ОДРЕДИТИ ВЕЛИЧИНУ ПРОБНОГ ОПТЕРЕЂЕЊА.

СТАЊЕ	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>	M <sub>9</sub>
0	10654	11235	10787	09636	10211	12358	12001	11353	10269
2	10560	11045	10693	09543	10117	12233	11872	11614	10534



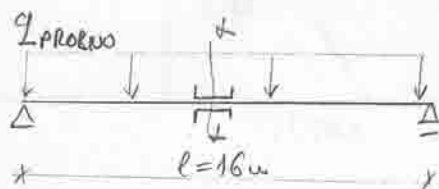
$$E_b = 0,3 \cdot 10^4 \text{ kN/cm}^2$$

$$E_c = 2,1 \cdot 10^4 \text{ kN/cm}^2$$

$$K_i = 2,10$$

$$K_t = 2,03 \text{ (ЗА БЕТОН)}$$

$$K_t = 1,97 \text{ (ЗА ЧЕЛИК)}$$



	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>	M <sub>9</sub>
Δ	-94	-190	-94	-93	-94	-125	-129	261	265
[x10 <sup>6</sup> ] E	-97,24	-196,55	-97,24	-96,207	-97,241	-133,2	-137,5	278,2	282,5
ε <sub>sa</sub>	-146,895 · 10 <sup>-6</sup>			-96,724 · 10 <sup>-6</sup>		-135,30 · 10 <sup>-6</sup>		280,3 · 10 <sup>-6</sup>	
ε <sub>sn</sub>		-0,441		-0,29		-2,84		5,89	

$$\epsilon = \frac{K_i}{K_t} \cdot \rho \cdot A_c \quad \rho = 10^{-6}$$

• ГЕОМЕТРИЈСКЕ КАРАКТЕРИСТИКЕ ПРЕСЕКА

$$A_b = 200 \cdot 8 = 1600 \text{ cm}^2 \quad A_{b1} = \frac{E_b}{E_c} \cdot A_b = \frac{1}{7} \cdot 1600 = 228,57 \text{ cm}^2$$

$$y_{TB} = 4 \text{ cm}$$

$$A_c = 2 \cdot 40 \cdot 3 + 54 \cdot 1,2 = 304,8 \text{ cm}^2 \quad y_{Tc} = 8 + 3 + \frac{1}{2} \cdot 54 = 38 \text{ cm}$$

$$I_b = 2 \cdot \frac{1}{12} \cdot 40 \cdot 3^3 + \frac{1}{12} \cdot 1,2 \cdot 54^3 + 2 \cdot 120 \cdot 28,5^2 = 210866,40 \text{ cm}^4$$

$$I_c = \frac{1}{12} \cdot 200 \cdot 8^3 = 8533,3 \text{ cm}^4 \quad I_{b1} = \frac{1}{7} \cdot I_b = 1219,05 \text{ cm}^4$$

$$A_i = A_{b1} + A_c = 228,57 + 304,8 = 533,37 \text{ cm}^2$$

$$y_{Ti} = \frac{228,57 \cdot 4 + 304,8 \cdot 38}{533,37} = 23,4 \text{ cm (ОД ГОРЊЕ УВУЧЕ)}$$

$$I_i = 1219,05 + 210866,40 + 228,57 \cdot (23,4 - 4)^2 + 304,8 \cdot (38 - 23,4)^2 = 363081,23 \text{ cm}^4$$

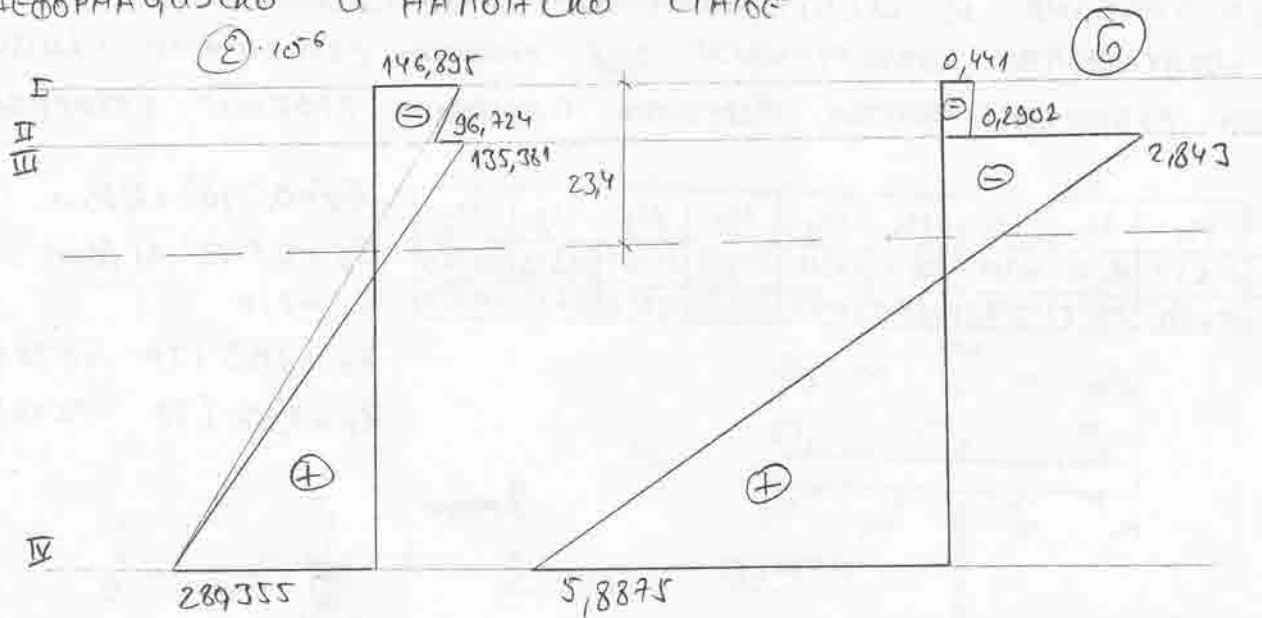
$$W_{Ti} = \frac{363081,23}{23,4} = 15516,29 \text{ cm}^3$$

$$W_{Ti} = \frac{363081,23}{44,8} = 8140,83 \text{ cm}^3$$

$$W_{Ti} = W_{Ti} = \frac{363081,23}{15,4} = 23576,86 \text{ cm}^3$$



# • ДЕФОРМАЦИЈСКО И НАПОНСКО СТАЊЕ



ИЗ ДИЈАГРАМА ДЕФОРМАЦИЈА СЕ ВИДИ ДА НАРЕЊЕ У ПРЕСЕКУ III ОДСТУПА ОД ОСТАЛИХ НАРЕЊА, КОЈА СЕ УКИДАЈУ У ХИПОТЕЗИ О РАВНОМ ПРЕСЕКУ ПРЕ И ПОСЛЕ ДЕФОРМАЦИЈЕ

• ОДРЕЂИВАЊЕ ПОДНОГ ОПТЕРЕЋЕЊА

$$M = \frac{1}{8} \cdot q \cdot l^2 \quad G = \frac{M}{W} = \frac{q \cdot l^2}{8 \cdot W}$$

$$q = \frac{8 \cdot W \cdot G}{l^2}$$

$$q_I = \frac{8 \cdot 15516,29 \cdot (0,441 \cdot 7)}{16^2 \cdot 10^4} = 0,15 \text{ kN/cm} = 15 \text{ kN/m}$$

$$q_{II} = \frac{8 \cdot 23576,86 \cdot (0,29 \cdot 7)}{16^2 \cdot 10^4} = 0,15 \text{ kN/cm} = 15 \text{ kN/m}$$

$$q_{III} = \frac{8 \cdot 23576,86 \cdot 2,84}{16^2 \cdot 10^4} = 0,21 \text{ kN/cm} = 21 \text{ kN/m} \rightarrow \text{ПОТРЕБНО НАРЕЊЕ}$$

$$q_{IV} = \frac{8 \cdot 8140,83}{16^2 \cdot 10^4} = 0,15 \text{ kN/cm} = 15 \text{ kN/m}$$

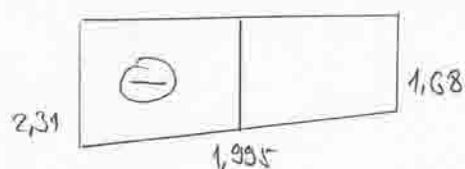
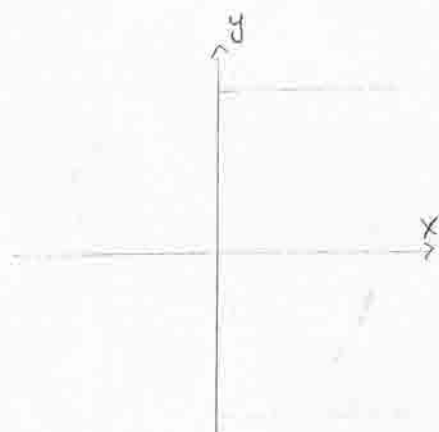
$$\boxed{q = 15 \text{ kN/m}}$$

НА ЧЕЛУЧНОМ СТУБУ УСПУЖЕНОМ БЕТОНОМ ПЕРЕЧЕ СЪ ЛОКАЛНЕ ДЕФ.  
ОПРЕДЕЛИТИ ПРЕСЕЧНЕ СИЛЕ + ПОБЕЖДЕРЖ

СТ.	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>K</sub>
О	0258	0333	0605	0281	0111
Р	0248	0321	0593	0292	0110
ΔЗ	-10	-12	-12	-9	-1
ΔЗ-ΔК	-9	-11	-11	-8	/
Е	-90	-110	-110	-80	/
Г	-1,89	-2,31	-2,31	-1,68	/

$$\varepsilon = \rho_{pf} \cdot \frac{K_i}{K_t} \cdot (\Delta \varepsilon - \Delta K)$$

$$G = \varepsilon \cdot E_i$$

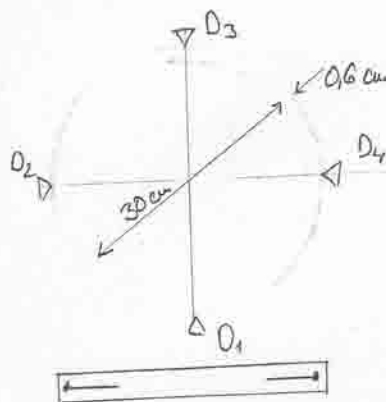


$$\rho_{pf} = 100 \text{ мм}$$

$$E_z = 2,10 \cdot 10^4 \text{ кН/см}^2 \quad \sqrt{\varepsilon} = 0,28$$

$$E_B = 0,30 \cdot 10^4 \text{ кН/см}^2 \quad \sqrt{\varepsilon} = 0,17$$

$$\rho_{pf} = 10 \cdot 10^{-6} \text{ мм/мм}$$



$$E_i = E_z = 2,10 \cdot 10^4 \text{ кН/см}^2$$

$$\frac{E_B}{E_z} = \frac{1}{7}$$

$$A = \frac{(31,2^2 - 30^2) \cdot \pi}{4} + \frac{E_B}{E_z} \cdot \frac{30^2 \cdot \pi}{4} = 158,66 \text{ см}^2$$

$$I_i = \frac{\pi}{64} [(31,2^4 - 30^4) + \frac{E_B}{E_z} \cdot 30^4] = 12433,821 \text{ см}^4$$

$$G_{Nx} = -2,10 \text{ кН/см}^2$$

$$G_{Mx} = \pm 0,21 \text{ кН/см}^2$$

$$G_{Ny} = -1,995 \text{ кН/см}^2$$

$$G_{My} = \pm 0,315 \text{ кН/см}^2$$

$$N = G_{Nx} \cdot A_i = -2,10 \cdot 158,66 = -333,186 \text{ кН}$$

$$N = G_{Ny} \cdot A_i = -1,995 \cdot 158,66 = -316,530 \text{ кН}$$

$$M_x = G_{Mx} \cdot W_i = 0,21 \cdot 797,040 = \pm 167,378 \text{ кН см}$$

$$M_y = G_{My} \cdot W_i = 0,315 \cdot 797,040 = \pm 251,070 \text{ кН см}$$

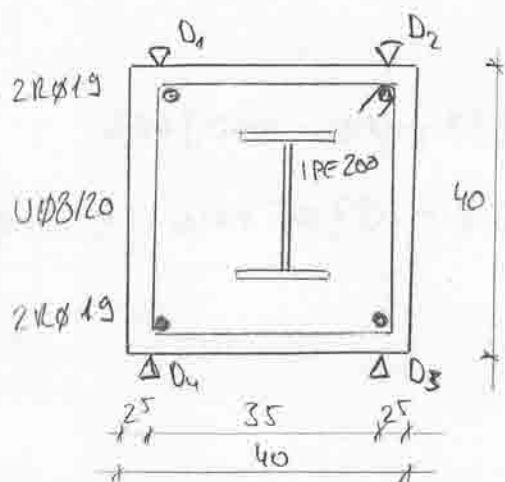
$$N = 0,5 \cdot (333,186 + 316,530) = 324,86 \text{ кН}$$

$$N = 324,86 \text{ кН}$$

$$M = \sqrt{167,378^2 + 251,070^2} = 301,75 \text{ кН см}$$

$$M = 301,75 \text{ кН см}$$

НА КОМПОЗИТНОМ СТУБѢ (У АБ СТѢ УБЕТОНИРАН ЧЕЛИЧНИ И НОСАЧ СУ, ПРЕМА СКИЦИ МЕРЕЊЕ ДИМЕНЗИЈАХ. ОДРЕДИТИ ПРЕСЕЧНЕ СИЛЕ.



$$E_s = 2,1 \cdot 10^4 \text{ kN/cm}^2 \quad \nu_s = 0,30$$

$$E_c = 0,35 \cdot 10^4 \text{ kN/cm}^2 \quad \nu_c = 0,18$$

$$A_{s1} 4 \cdot \varnothing 19 = 2,84 \text{ cm}^2$$

$$A_{s2} \varnothing 8 = 0,50 \text{ cm}^2$$

$$A_{IPE 200} = 28,5 \text{ cm}^2$$

$$h = 200 \text{ mm} \quad b = 100 \text{ mm} \quad s = 5,6 \text{ mm} \quad t = 8,5 \text{ mm}$$

$$I_x = 1940 \text{ cm}^4 \quad I_y = 14 \text{ cm}^4$$

С.Т	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>
О	0258	0333	0605	0281	0111
P	0248	0321	0593	0272	0110
P-O	-10/-9	-12/-11	-12/-11	-9/-8	-1/-
[ $\times 10^6$ ]	E	-90	-110	-110	-80
[kN/cm <sup>2</sup> ]	G	-0,315	-0,395	-0,385	-0,280

ИНСТРУМЕНТ ПОКРЕЊЕЊ  $\ell = 100 \text{ mm}$

D<sub>ik</sub> - ЧИТАЊЕ НА КОМПЕНЗАЦИОНОМ  
СТАЊУ

ПОКРЕЊЕЊ  $\ell = 100 \text{ mm}$

$$E_i = E_c = 0,35 \cdot 10^4 \text{ kN/cm}^2$$

• ГЕОМЕТРИЈСКЕ КАРАКТЕРИСТИКЕ ПОПРЕЧНОГ ПРЕСЕКА

$$n = \frac{E_s}{E_c} = \frac{2,1}{0,35} = 6$$

$$A_i = A_c + (n-1) \cdot A_s = 40 \cdot 40 + (6-1) \cdot (4 \cdot 2,84 + 28,5)$$

$$A_i = 1600 + 5 \cdot 39,86 = 1799,3 \text{ cm}^2$$

$$I_{ix} = I_{cx} + (n-1) I_{sx} = \frac{1}{12} \cdot 40^4 + (6-1) (1940 + 4 \cdot 2,84 \cdot 15^2) =$$

$$= 213333,3 + 5 \cdot (1940 + 2556) = 235813,33 \text{ cm}^4$$

$$W_{ix}^{\max} = \frac{I_{ix}}{20} = 11790,67 \text{ cm}^3$$

$$I_{iy} = I_{cy} + (n-1) I_{sy} = \frac{1}{12} \cdot 40^4 + (6-1) (142 + 4 \cdot 2,84 \cdot 15^2) =$$

$$= 213333,3 + 5 \cdot (142 + 2556) = 226823,33 \text{ cm}^4$$

$$W_{iy} = \frac{I_{iy}}{20-2,5} = 12961,33 \text{ cm}^3$$

• APF REYHE CUMEN

$$G_x^{sr} = G_y^{sr} \rightarrow N_x = N_y = N = G^{sr} \cdot A_i$$

$$N = -0,34125 \cdot 1799,3 = -614,01 \text{ kN}$$

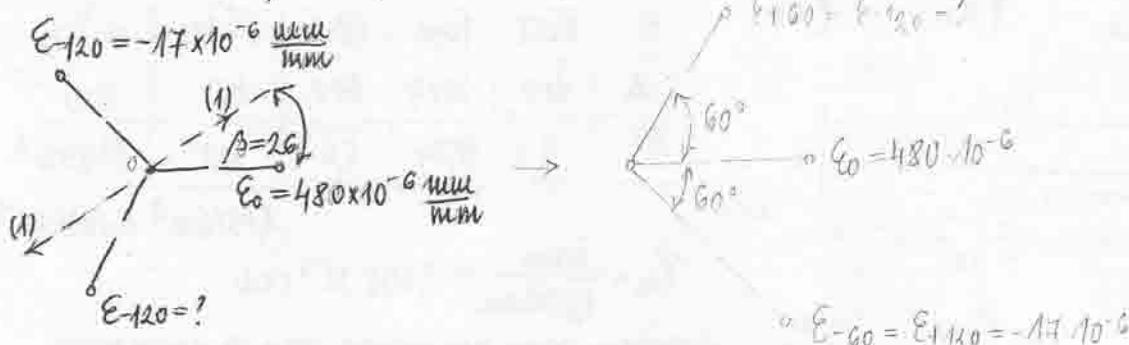
$$M_x = \Delta G_x \cdot W_{ix} = 0,00875 \cdot 11790,67 = 103,17 \text{ kNm} = 103 \text{ kNm}$$

$$M_y = \Delta G_y \cdot W_{iy} = (0,385 - 0,34125) \cdot 12961,33 = 567,06 \text{ kNm} = 5,67 \text{ kNm}$$

JUN 2002 - 2

- ① NA ČELIČNOM ZEMENITU MERENJE SU LOKALNE DEFORMACIJE PUTEM ROZETE. MERENJE U PRAVCU  $\epsilon_{-120}$  JE BILO SPREČENO, TE JE IZVRŠENO MODELIRANJE TOG KONSTRUKTIVNOG ELEMENTA U STROGOJ GEOMETRIJSKOJ SLIČNOSTI ZA NAPONSKU OPTSKU ANALIZU. NA MODELU SU U POSMATRANOJ TAČKI ODRŽANI REZ IZDIRKOM  $n = 14,5$  I UGLAD KRSTA GLAVNIH NAPONA  $\beta \approx 26^\circ$  U PRAVCU SUPROTNO OD KAZALKE NA SATU. ODRŽITI GLAVNE DILATACIJE I NAPONE.

MODEL:  $E_m = 7000 \text{ kN/cm}^2$ ;  $d_m = 0,5 \text{ cm}$ ;  $\alpha = 0,115 \text{ kN/cm rez}$



$$\epsilon_{1,2} = \frac{\epsilon_{60} + \epsilon_{-60} + \epsilon_0}{3} \pm \frac{1}{3} \sqrt{(2\epsilon_0 - \epsilon_{60} - \epsilon_{-60})^2 + 3(\epsilon_{60} - \epsilon_{-60})^2}$$

$$+ \tan 2\alpha^* = \left| \frac{\epsilon_{60} - \epsilon_{-60}}{2\epsilon_0 - \epsilon_{60} - \epsilon_{-60}} \right| \sqrt{3}$$

$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2), \quad \sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1)$$

$$\sigma_1 - \sigma_2 = \frac{E}{1-\nu^2} [\epsilon_1(1-\nu) - \epsilon_2(1-\nu)] = \frac{E}{1+\nu} (\epsilon_1 - \epsilon_2)$$

$$N_b (\sigma_1 - \sigma_2)_m = (\sigma_1 - \sigma_2)_p \rightarrow \text{STROGA GEOMETRIJSKA SLIČNOST}$$

$$n = \frac{d}{b} (\sigma_1 - \sigma_2)_m \rightarrow (\sigma_1 - \sigma_2)_m = \frac{n \cdot c}{d} = \frac{14,5 \cdot 0,115}{0,5} = 3,335$$

$$(\epsilon_1 - \epsilon_2)_p = \frac{1+\nu}{E} (\sigma_1 - \sigma_2)_p = \frac{1,3}{2,1 \cdot 10^4} \cdot 3,335 \cdot \frac{2,1}{7,0} = 619,357 \cdot 10^{-6}$$

$$\alpha_{op} \approx 26^\circ$$

$$\epsilon_1 - \epsilon_2 = \frac{2}{3} \sqrt{(2\epsilon_0 - \epsilon_{120} - \epsilon_{120})^2 + 3(\epsilon_{120} - \epsilon_{120})^2} = 619,357 \cdot \frac{3}{2}$$

$$(977 - \epsilon_{120})^2 + 3 \cdot (-17 - \epsilon_{120})^2 = 863106,9603$$

$$954529 - 1954 \cdot \epsilon_{120} + \epsilon_{120}^2 + 867 + 102 \epsilon_{120} + 3 \epsilon_{120}^2 = 863106,9603$$

$$4 \epsilon_{120}^2 - 1852 \epsilon_{120} + 92289,0397 = 0$$

$$\epsilon_{120}^2 - 463 \epsilon_{120} + 23072,25993 = 0$$

$$(\epsilon_{120})_{1,2} = \frac{463 \pm 349,399}{2}$$

$$(\tan 2\alpha^*)_1 = \left| \frac{-423,2}{570,8} \right| \sqrt{3} = 7 \quad \alpha^* = 26,046^\circ$$

$$\epsilon_{120} = 406,2 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\epsilon_{120}^1 = 406,2$$

$$\epsilon_{120}^2 = 56,8005$$

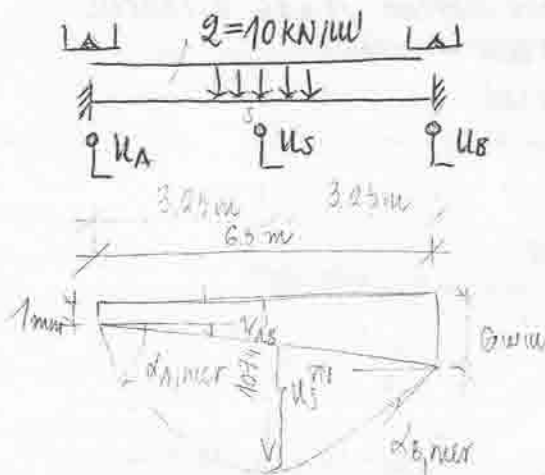
$$\epsilon_1 = 599,412 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\sigma_1 = 13,234 \text{ kN/cm}^2$$

$$\epsilon_2 = -19,946 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$\sigma_2 = 3,689 \text{ kN/cm}^2$$

②③ PRILIKOM ISPITIVANJA ČELIČNE GREDE DOBIJENI SU REZULTATI DATI U TABELI NAČI STVARNE OSLONAČKE MOMENTE  $M_A$  I  $M_B$ , KAO I STEPENI UKLOSTENJA I UPOREDITI SA RAČUNSKIM  $\eta_A = 0,01 \text{ mm}$ ,  $I_{x-x} = 4250 \text{ cm}^4$ ;  $E = 2,1 \times 10^4 \text{ kN/cm}^2$ .  $\rho = 0,01 \text{ mm}$



STANJE	$U_A$	$U_S$	$U_B$	$K_1$	$K_2$
0	0158	0025	0223	0+023	0+055
2	0258	1099	0823	0+123	0+173
$\Delta$	100	1074	600	350	132
$W$	1,0	1074	6,0	371"	13992"

$$1,7986 \cdot 10^{-3} \quad 6,7835 \cdot 10^{-4} \quad \alpha \text{ (rad)}$$

$$\gamma_{AB} = \frac{5 \text{ mm}}{6500 \text{ mm}} = 7,692 \cdot 10^{-4} \text{ rad}$$

(OBRATNI STEPENI UKLOSTENJA OD UJEDNAKOVANOG SLABANJA OSNOVNA)

$$\alpha_{A,SN} = \alpha_{A,mer} - \gamma_{AB} = (1,7986 \cdot 10^{-3} - 7,692 \cdot 10^{-4}) = 1,0294 \cdot 10^{-3}$$

$$\alpha_{B,SN} = \alpha_{B,mer} + \gamma_{AB} = (6,7835 \cdot 10^{-4} + 7,692 \cdot 10^{-4}) = 1,44755 \cdot 10^{-3}$$

MOMENTI (STVARNI):

$$\left. \begin{aligned} M_A &= \frac{2EI}{l} [(\alpha_B - 2\alpha_A) + 3\gamma_{AB}] + \frac{2l^2}{12} \\ M_B &= \frac{2EI}{l} [(\alpha_A - 2\alpha_B) - 3\gamma_{AB}] + \frac{2l^2}{12} \end{aligned} \right\} + \text{MOMENT AKO ŽATJE GORNJE VLAKE}$$

$$\frac{2EI}{l} = 2 \cdot \frac{2,1 \cdot 10^4 \cdot 4250}{650} = 274615,3 \text{ kN cm} = 2746,153 \text{ kNm}$$

$$M_{A,SN} = 2746,153 \cdot [(6,7835 \cdot 10^{-4} - 2 \cdot 1,7986 \cdot 10^{-3}) + 3 \cdot 7,692 \cdot 10^{-4}] + \frac{10 \cdot 6,5^2}{12}$$

$$= (1,44755 \cdot 10^{-3} - 2 \cdot 1,0294 \cdot 10^{-3})$$

$$M_{A,SN} = 39,870 \text{ kNm}$$

$$M_{B,SN} = 2746,153 \cdot [(1,0294 \cdot 10^{-3} - 2 \cdot 1,44755 \cdot 10^{-3}) - 3 \cdot 7,692 \cdot 10^{-4}] + \frac{10 \cdot 6,5^2}{12}$$

$$M_{B,SN} = 23,747 \text{ kNm}$$

STEPENI UKLOSTENJA:  $\alpha_{A,B}^{rac} = \frac{2l^2}{12 \left( \frac{2EI}{l} \right)} = \frac{10 \cdot 6,5^2}{12 \cdot 2746,153} = 12,8209 \cdot 10^{-3} \text{ rad}$

$$\eta_A = \left( 1 - \frac{\alpha_{A,SN}}{\alpha_{A,rac}} \right) \cdot 100\% = \left( 1 - \frac{1,0294 \cdot 10^{-3}}{12,8209 \cdot 10^{-3}} \right) \cdot 100\% = 91,97\%$$

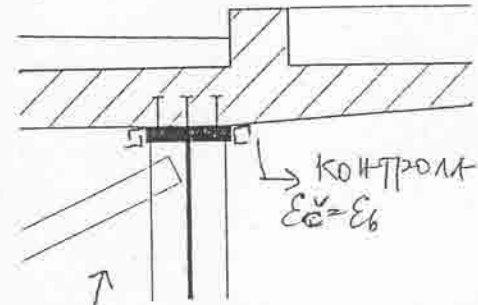
$$\eta_B = \left( 1 - \frac{\alpha_{B,SN}}{\alpha_{B,rac}} \right) \cdot 100\% = \left( 1 - \frac{1,44755 \cdot 10^{-3}}{12,8209 \cdot 10^{-3}} \right) \cdot 100\% = 88,72\%$$

40

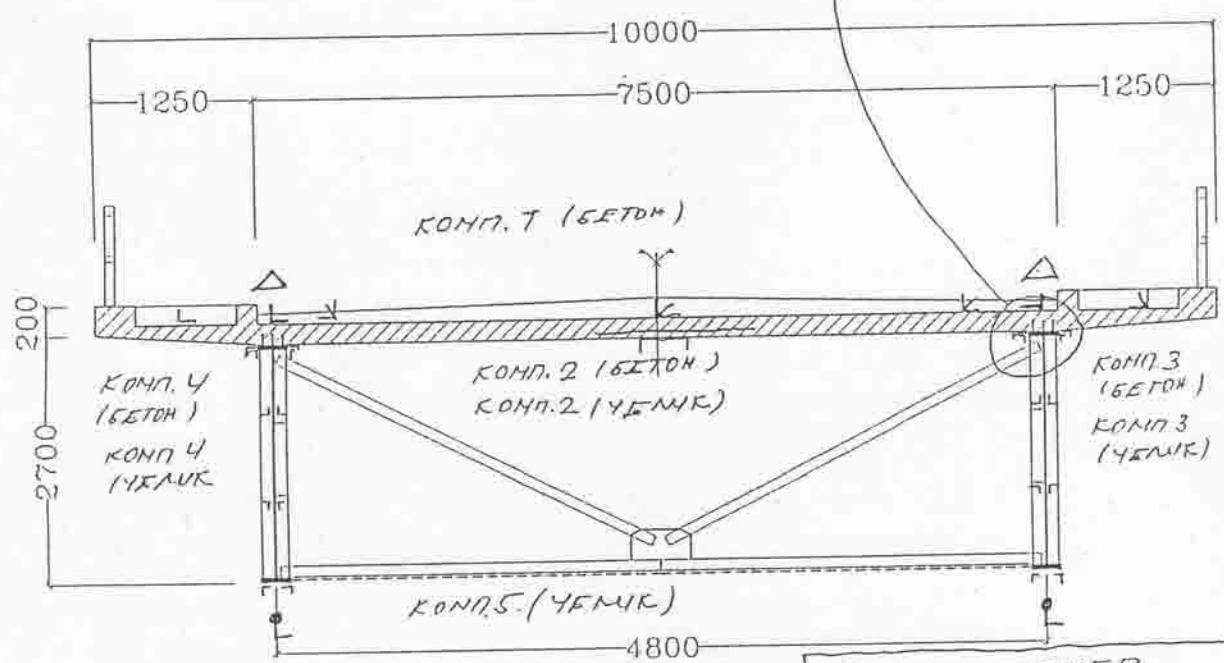
4. Na prikazanoj konstrukciji mosta dati detaljan raspored mernih mesta za merenje opštih i lokalnih deformacija, kao i raspored i broj kompenzacionih mernih mesta.

опште гер.  $5+2=7$  (уг. + ка.)  
 локалне гер.  $8(п.ч.) + 7(п.от.) + 3(деформ.) = 18$   
 комп. м.  $5$

(30)



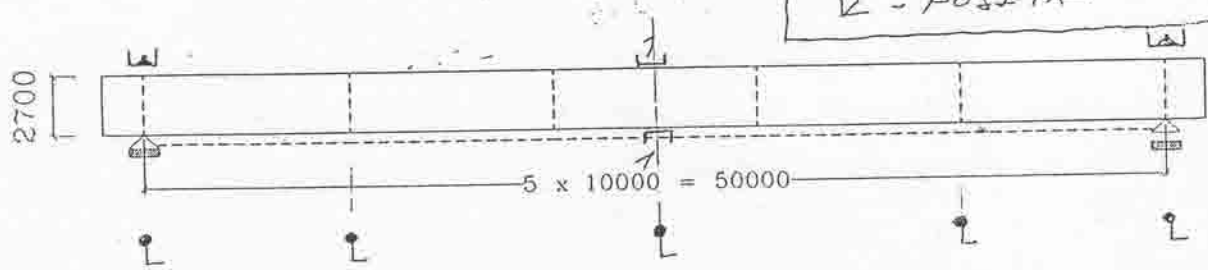
poprečni presek mosta:



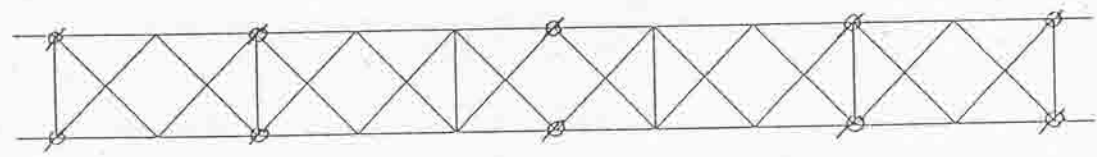
1-1

Ø - УГЛОМЕР  
 Δ - КЛИНОМЕТАР  
 L - МЕРНА ТРАКА  
 K - РОЗЕТА

glavni nosač:



спрег за ветар:

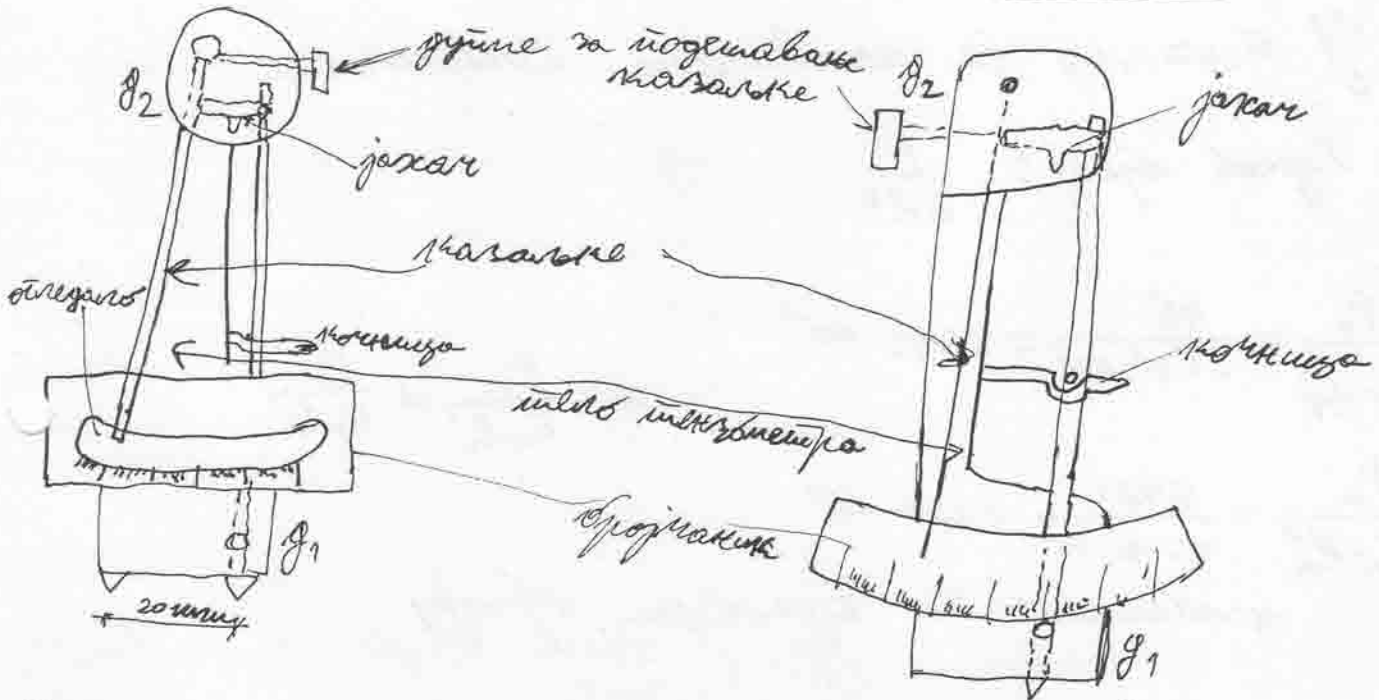




2) 31.08.2006. Дати порачају између титанометра  
типа "Хукенбергер" и "Галилео" (дати сличности и разлике). Објаснити принцип на  
коме ради титанометар и објаснити вентил који  
принцип на вентилу мерне базе.

"ХУКЕНБЕРГЕР"

"ГАЛИЛЕО"



⊗ Сличности: — тело у облику слова "L"

- лачни бројчаник са 40 подова
- принцип уветања преко покретних палца (покретна ножица, жакан, капаљка, вентил  $d_1$  и  $d_2$ )
- одна прамена палца понов да је  $U=1000$
- имају заштитну плочу  $d_2$  и вентил за подизање капаљке

⊗ РАЗЛИКЕ:

- Хукенбергер има основну базу од 20mm, Галилео нема
- Хукенбергер у равни бројчаника ~~има~~ <sup>има</sup> огледало ради тачније и лакше читаности.
- Галилео има могућност прачене уветања 1000/200

$$N_P^{-1} = \frac{10 \cdot 9,3}{9986,325} \Rightarrow N_P = 107 = \frac{P_P}{P_m}$$

$$P_m = \frac{P_P}{107} = \frac{45}{107} = 0,421 \text{ KN}$$

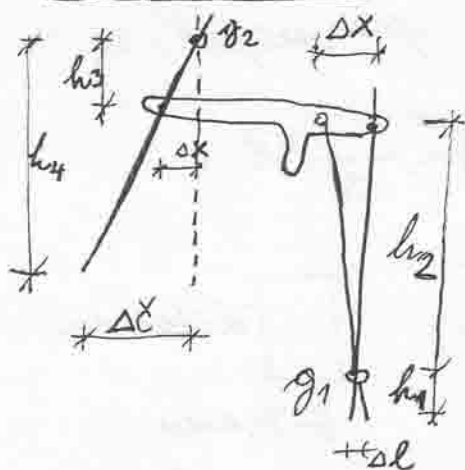
- У питању је водична стичност

- Хукс брџ:  $\frac{P}{EL^2}$

$$\left. \begin{aligned} \frac{P_P}{E_P \cdot l_P^2} &= \frac{45}{2,1 \cdot 10^9 \cdot 10^4} = 21,43 \cdot 10^{-8} \\ \frac{P_m}{E_m \cdot l_m^2} &= \frac{0,421}{0,7 \cdot 10^4 \cdot 10^2} = 600 \cdot 10^{-8} \end{aligned} \right\} \Rightarrow \frac{P_m}{E_m l_m^2} \neq \frac{P_P}{E_P l_P^2}$$

водичне по Хуковом брџу

# - ПРИНЦИП РАБА ТЕНЗОМЕТРА



померяване жаксара:

$$\frac{\Delta l}{h_1} = \frac{\Delta X}{h_2} \Rightarrow \Delta X = \Delta l \cdot \frac{h_2}{h_1}$$

$$\frac{\Delta \check{C}}{h_4} = \frac{\Delta X}{h_3} \Rightarrow \frac{\Delta \check{C}}{h_4} = \Delta l \cdot \frac{h_2}{h_1 \cdot h_3}$$

проблемна джуксара баже меркжа:

$$\Delta l = \frac{\Delta \check{C}}{\frac{h_2 \cdot h_4}{h_1 \cdot h_3}} = \frac{\Delta \check{C}}{U_T}$$

$$U_T = \frac{h_2}{h_1} \cdot \frac{h_4}{h_3} = \frac{120 \cdot 100}{3 \cdot 4} = 10000 \quad - \text{убетаксе мет самекатжа}$$

$$\varepsilon = \frac{\Delta l}{l} = \frac{\Delta \check{C}}{U_T \cdot l}$$

$$\mu_T = \varepsilon_{(1)} = \frac{1}{1000 \cdot \underset{20 \text{ mm}}{l}} = \frac{1}{2 \cdot 10^4} = 50 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}}$$

$$l = 100 \text{ mm} \quad \mu_T = 10 \cdot 10^{-6} \text{ mm/mm}$$

смететин кжа гжмжж ко блмжжж  
мерже баже:

$l = 20 \text{ mm}$  - приметаксва самс ко чмжжж  
(за хомоксн матермжжж)

$l = 100 \text{ mm}$  - чмжжж и блмжжж

$l = 1000 \text{ mm}$  - за кжмжжж